

THE ARITHMETIC TEACHER

Volume 8, number 1 JANUARY 1961

Mathematics as a cultural heritage *William L. Schaaf*

A case in point *Wayne Peterson*

Highlights of a summer conference *Catherine Linn Davis*

Problems without numbers *Walter L. Klas*

Testing—without tests *K. L. Harrison*

*Report of the MSG Elementary-School Mathematics project,
suggestions for teaching multiplication and division,
a review of mathematics articles in The World Book Encyclopedia,
and the report of the nominating committee*

A JOURNAL OF *The National Council of Teachers of Mathematics*

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About our editorial staff

E. GLENADINE GIBB *Editor*

This is the beginning of another year—the eighth for *THE ARITHMETIC TEACHER*. Since no one person single-handedly composes a magazine, in this, the first issue of the Eighth Volume, I wish to introduce those persons who have not only helped with the last three issues, but who will play major roles in the preparation of forthcoming issues. We hope that you will become better acquainted with them through the pages of *THE ARITHMETIC TEACHER*.

The pertinent editorial comments, "About the Articles," appearing in this issue as well as the "Editor's Note" after each article in the last three issues are written by our associate editor, Professor E. W. Hamilton, Iowa State Teachers College. He has taught in the rural schools of Nebraska, the secondary schools of Missouri, and has been connected with various phases of education in Iowa for the past twenty-odd years. Presently he is engaged in research on media of instruction suitable for larger college classes in mathematics. These experiences together with his fundamental concern for both the pre-service and in-service mathematics education of elementary teachers make him especially valuable in the development of editorial policy.

Edwina Deans, Clarence E. Hardgrove, and J. Fred Weaver, assistant editors, are primarily responsible for their indicated sections of each issue of *THE ARITHMETIC TEACHER*.



How many of you like to try a new idea in the classroom? Are you willing to share your ideas with others? "In the Classroom," Dr. Edwina Deans' monthly feature, will be devoted to suggestions for teaching. Dr. Deans, a consultant in elementary education for the Arlington, Virginia, Public Schools, is herself a creative teacher and has written *Arithmetic—Children Use It* and *The Three R's in the Elementary School*. Since Dr. Deans has conducted many workshops and in-service programs, those of you in Florida, Virginia, and Michigan are already acquainted with her.

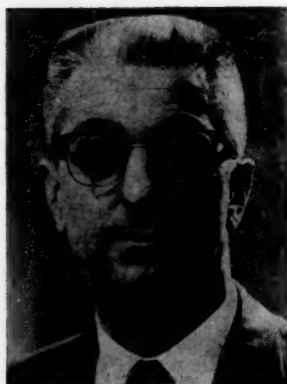
Reporting "Experimental Projects and Research" is another new venture in the past three issues and one we hope to continue. This department contains information regarding the many experimental programs in elementary-school mathematics. J. Fred Weaver, professor of mathematics education, director of graduate studies, associate dean and chairman of the department of education at Boston University School of Education, is editor of this department. He is actively asso-



Edwina Deans
"In the Classroom"



Clarence E. Hardgrove
"Reviews"



J. Fred Weaver
*"Experimental Projects
 and Research"*

ciated with the work of the elementary-school mathematics division of the School Mathematics Study Group, serving as member of the advisory panel, as member of the steering committee for the writing group, and as chairman-consultant for the 1960-61 Boston Center.

Each month in "Reviews," we shall continue to evaluate new materials as they become available. Clarence Ethel Hardgrove, editor of this section, is a professor of mathematics at Northern Illinois University and presently is vice-president (elementary) of the National Council of Teachers of Mathematics. Dr. Hardgrove is an active member of the Illinois Council of Teachers of Mathematics, is coauthor of *Thinking in the Language of Mathematics* (Illinois Curriculum Program), and many of you have heard her speak at mathematics meetings. She is the author of the NCTM publication, *Elementary and Junior High School Library*, and has contributed chapters to the Twenty-second and Twenty-fifth Yearbooks of the National Council of Teachers of Mathematics. Professor Hardgrove was a member of the 1960 SMSG Elementary-School Mathematics writing group.

The editor and the three assistants pictured on page 3 consider all submitted manuscripts and make decisions regarding those to be accepted for publication. If you have an idea for a manuscript in mind or a contribution to a particular department, put it on paper and send it to one of these staff members.

Those of you in the West, in particular, know Professor Marguerite Brydegaard, the editor's right hand in securing manuscripts. Dr. Brydegaard is well qualified to accept editorial responsibilities. She has been a member of THE ARITHMETIC TEACHER editorial staff since 1956. She has been a member of the editorial board of *Childhood Education International* and is presently a member of the supplementary publications committee of the National Council of Teachers of Mathematics and also a columnist for *The In-*

structor. A professor of education at San Diego State College, Dr. Brydegaard has written articles, developed tests, and prepared films. You can learn more about her in *Who's Who of American Women* and *Who's Who in the West*.

Although he could qualify for a well-earned vacation from professional responsibilities, Dr. John R. Clark has been willing to continue to serve readers of *THE ARITHMETIC TEACHER* as a member of the editorial staff. We are indeed grateful for the good judgment and sage advice of this elder statesman of elementary-school mathematics in our policy-making decisions. As a teacher in rural schools of Indiana, secondary schools of Chicago, and as professor of education at Teachers College, Columbia University, he has had an active professional career. Many children, teachers, and members of professional organizations have profited by his fine contributions to elementary-school mathematics. Dr. Clark now serves as consultant in mathematics education and resides in New Hope, Pennsylvania.

Those of us who have been particularly interested in research in arithmetic need no introduction to our assistant editor Professor Vincent J. Glennon, who wrote, *What Does Research Say About Arithmetic?* He is also a member of the editorial board for the forthcoming NCTM yearbook on *Mathematics for the Talented*. He is director of the Arithmetic Studies Center, Syracuse University. He is also presently serving as chairman of the Arithmetic Committee of the Association of Mathematics Teachers of New York State. His teaching responsibilities have not been limited to Syracuse but include Teachers College, Columbia, New York University, Ohio University, San Francisco State College, Harvard, and St. Johns College in Cleveland, Ohio.

Your editor is grateful for her good fortune in securing the assistance of these fine people who have been willing to add specific responsibilities for *THE ARITHMETIC TEACHER* to their already heavy professional schedules.



Marguerite Brydegaard
"Have you
something to write?"



John R. Clark
"Write it!"



Vincent J. Glennon
"Send it to us!"

About the articles

E. W. HAMILTON *Associate Editor*

The articles in this issue deal with several aspects of the modernizing and mathematizing process now gaining momentum in arithmetic instruction. Davis' report of a suggested sequence for the training of elementary-mathematics teachers should make us sit up and take notice. How many present teachers could qualify? How many institutions offer such a program? We seem headed toward special teachers in arithmetic. Do people realize and accept this?

This suggested sequence doesn't mention most of the things teachers remember learning in grade school nor does it mention most of the topics important to banking, commerce, and economic competence. It is "modern" in flavor and hence, paradoxically, is concerned with foundations and generalities rather than with applications and specific problems. Much of the modernizing of mathematics consists of increasing the precision of the language and the discovery and recognition of the logical foundations underlying it. Certainly little harm and probably much good will come from making sure that our early presentations to children are precise as far as they go. Articles by Peterson and Rapaport both touch on this question of precision of terms, and, whether you agree or not, they raise one of the issues—the "language clean-up"—long overdue.

Since we seem to be tending toward deductive systems and the study of number for number's sake, we find that Osborn's and Klas's articles on the nature and place of models, the relating of symbols to mod-



els, and the interpretation of symbols back into the physical situation, are timely. A problem seldom, if ever, has any numbers associated with it until we recognize the situation and employ numbers to describe it. This point of view cuts both ways and justifies our concern with models and oral arithmetic as well as our concern with generalized knowledge about numbers. We have to be able to think well enough to arrange the elements of a situation and adopt a strategy using models from reality, concepts, mental imagery, or whatever you prefer, but we also have to know enough about numbers to recognize the one or several properties that fit the situation, thus choosing an abstract model in the best mathematical sense.

The issue is rounded out by Schaaf's review of the history and development of mathematics, the latter part of which particularly will enlighten many teachers as to the ancient origin of some of our newest concerns and explain why mathematicians are dissatisfied with utilitarian, grocery-store arithmetic as a youngster's *chief diet*.

Mathematics as a cultural heritage

WILLIAM L. SCHAAF *Brooklyn College, Brooklyn, New York*

Dr. Schaaf is professor of education at Brooklyn College.

Broadly speaking, major cultures can be identified, at least in part, by certain outstanding characteristics. Thus, Babylonia and Egypt were steeped in mysticism and sensuality; the Greeks were preoccupied with ideas and ideals; the Romans, with politics, military prowess, and conquest. The culture of Western Europe from 600 A.D. to 1100 A.D. was expressed largely by its theology. From 1200 to 1800 it was the exploration of nature and the beginnings of science that marked the essence of the period. The spirit of the nineteenth and twentieth centuries, unless it is too early to judge in proper perspective, is typified by man's increasing mastery over his physical environment. This is evidenced not only by the general achievements of science and technology, but also by unprecedented industrial production, effective mass communication, and increasing automation. The creative language of the culture of today is science, and mathematics is the alphabet of science.

Contributions from the age of empiricism in mathematics

In past ages, mathematics was very largely a tool that not only facilitated the development of a culture, but which was itself more or less shaped by the culture. The pre-Grecian period may be aptly characterized as an age of empiricism in mathematics. Babylonian and Egyptian mathematics were concerned chiefly with astronomical and calendar questions, with the construction of tombs, temples and other religious buildings, and with prac-

tical problems of land measurement, surveying, and primitive engineering. In fact, the knowledge of arithmetic and mensuration possessed by the Babylonians appears to have been derived from the earlier work of the Sumerians who preceded them. By 2500 B.C., merchants of Sumer were already thoroughly acquainted with weights and measures, and were using the arithmetic of simple and compound interest with more than ordinary zeal. The Babylonians were prolific in the creation of elaborate multiplication tables and tables of squares and square roots. They may even have used the zero, although this invention is generally attributed to the Hindus. In short, the Babylonians were very skillful computers.

The Egyptians, whose mathematics was similarly empirical, were likewise adept at calculation. As early as 3500 B.C., they had extended their use of numbers to include hundreds of thousands and millions. Since there was neither inflation nor a national debt, they presumably had little need for billions. The Egyptians revealed an amazing ingenuity in their use of unit fractions; they were aware of the value of checking computation; and there is reason to believe that they anticipated the generalized number concept by using negative numbers as numbers.

Despite the fact that the mathematics of Babylon and Egypt were basically empirical, these two cultures nevertheless left their stamp upon the future in several ways. The idea of number was pressed into service for the market place as well as for

the contemplation of the heavens; the idea of geometric form was embraced in practical measurement in surveying, engineering, and astronomy; a distinct if feeble beginning was made in the use of algebraic symbolism; the generalized extension of the system of natural numbers was at least anticipated if not consciously fashioned; and out of experiences with measurement, there grew some awareness of the notion of the mathematical infinite.

The influence of Greek contributions and attitudes

With the ancient Greeks, some six centuries before the Christian era, mathematics came of age. For the next nine hundred years the contributions of Greek culture to mathematics were of the greatest significance, although oddly enough their influence upon arithmetic, as we know it at the present time, was little more than trivial.

It must be appreciated that the Greeks distinguished carefully between two aspects of knowledge about ordinary numbers: *logistiké*, the art of calculating, and *arithmetiké*, an abstract theory of numbers. *Logistica* (in the later Latin form) comprised the techniques of numerical computation in everyday trade and commerce, and in the arts and sciences, including geography and astronomy. *Arithmetica*, on the other hand, dealt with the properties of numbers as such, and in this sense was roughly comparable to contemporary "higher arithmetic" or the elementary theory of numbers (primes, factorization, congruences, etc.). In the eyes of the Greeks, to be concerned with *logistica* was considered beneath the dignity of mathematicians and philosophers, who assigned this drudgery to lesser persons, while they devoted their attention to *arithmetica*, geometry, and philosophy. It should be added that Greek *logistica*, compared to modern methods of computation, was exceedingly cumbersome and crude, due chiefly to an inadequate system of numeration.

These two aspects of the study of numbers were regarded separately until about the time of the invention of printing, although from time to time the names were changed. In the Middle Ages, *logistica*, or "practical arithmetic," was referred to by Italian writers as *practica* or *pratiche*; Latin writers of the Renaissance spoke of the art of computation as *ars supputandi*; Dutch writers called it "ciphering." We still sometimes speak of computing as reckoning, figuring, or calculating; the Germans use the term *Rechnen* for computation, although their term *Arithmetik* covers both aspects. Our modern word "arithmetic" began to be used for both branches in the early part of the sixteenth century.

Not much need be said here about Greek *arithmetiké*. To see it in appropriate perspective, it may be recalled that the Pythagoreans (500 B.C.) divided mathematical studies into four branches, the *quadrivium*: numbers absolute, or *arithmetica*; applied numbers, or *music*; magnitudes at rest, or *geometry*; and magnitudes in motion, or *astronomy*. The weakness of Greek arithmetic lay in the fact that almost from the very beginning, Greek mathematics succumbed to the number mysticism of the oriental cultures. As the centuries slipped by, the superstitious beliefs and esoteric lore associated with numbers increased to the detriment of Greek science and mathematics. Number "magic," *Gematria*, and other precursors of medieval and modern numerology were without doubt one of the facts which prevented the Greeks from embracing algebra and, possibly, even inventing the calculus.

Despite these shortcomings, however, during a period of nearly a thousand years, the Greeks did make several important contributions to arithmetic, or the theory of numbers; for example, (1) they arrived at certain basic theorems concerning the divisibility of numbers; (2) they discovered and proved that the number of primes is infinite; (3) they proved that $\sqrt{2}$ was irrational, and, in general, that the

side of a square is incommensurable with its diagonal.

The greatest cultural contribution of the Greeks to mathematics was not their insight into number theory, which, after all, was essentially elementary, nor their skill in the art of computing, which was inconsequential. Their contribution had to do rather with two fundamental concepts or attitudes. One was their faith in the method of deductive reasoning as a sound basis upon which to build the structure of geometry. The other was the belief that our physical environment could be described in mathematical terms; that, in short, number is the language of science. Both legacies were destined to exert a profound influence upon Western civilization for the next two thousand years.

A barren period

After the fall of Rome, Western Europe entered upon a period of stagnation and groping for nearly a thousand years. From about 400 to 1500 A.D., mathematics reflected the general cultural condition of the times—meagre and barren. Such as it was, this mathematics was kept alive by a handful of individual laymen and ecclesiastical scholars. One of the earliest influential laymen of this period was Boethius (c.500 A.D.). Among the ecclesiastical mathematicians were Alcuin (c.775), and the French monk Gerbert, who later became pope under the name of Sylvester II (c.1000). Gerbert traveled extensively in Italy and Spain, becoming familiar with Arabic mathematics—especially the Hindu-Arabic numerals.

With the decline of feudalism during the twelfth, thirteenth, and fourteenth centuries there emerged such powerful commercial city-states as Milan, Venice, Florence, Pisa, and Genoa. At the same time, mathematics began to feel the impact of advances in technology and crafts, as well as the effect of a rising trade and money economy. In particular, mathematics was influenced by architecture, military engineering, navigation, and astron-

omy on the one hand, and, to a lesser extent, by trading, accounts, and commercial activities. The latter involved barter, exchange, customs, drafts, interest, discount, usury, rents, annuities, insurance, partnerships, and stocks. An outstanding writer was Leonardo of Pisa, also known as Fibonacci, whose widely known book, *Liber Abaci* (1202), helped spread the Hindu-Arabic system of numeration in Western Europe, but not without considerable resistance.

Introduction of Hindu-Arabic numerals and a new interest in arithmetic

The Hindu-Arabic numerals had found their way into Europe originally through the contacts of traders with Moorish merchants, and through scholars who studied in Spanish universities. At first not well received, their adoption was greatly hindered by inertia and prejudice, and it was not until the sixteenth century that their numerals were in common use throughout Europe.

The fall of the Byzantine Empire, about 1450, saw a revival of interest in mathematics, particularly a study of the original Greek works. Scholarly activities had also moved northward into Central Europe. One of the most influential writers at this time, the German mathematician Regiomontanus, famous for his work in trigonometry, gave considerable impetus to the general interest in mathematics. During the two centuries following Fibonacci, the pace of progress had accelerated: printing had become a fact; the Hindu-Arabic numerals were beginning to take hold; and interest in mathematics was spreading beyond the Italian cities. Close upon the heels of the trigonometry of Regiomontanus came Luca Pacioli's *Summa de Arithmetica* (1494), one of the earliest and most-celebrated printed books on arithmetic. By now the Hindu-Arabic numerals were fairly well established.

Sixteenth-century arithmetic flourished, although new horizons in mathematics also

began to dawn. More and more scholars became interested in mathematics; textbooks became more plentiful; interest in science and mechanics increased; and mathematics was being studied for its own sake.

During this fruitful period we find Robert Recorde's *The Ground of Artes* (London, 1540), one of the most popular arithmetics ever printed; the Dutch writer Gemma Frisius, author of a popular text on combined theoretical and commercial arithmetic; Simon Stevin, the Flemish mathematician who was instrumental in furthering the general acceptance of decimal fractions; and John Napier, the aristocratic Scotch baron whose invention of logarithms (1614) was epoch-making. From 1650 to 1850, the interest of mathematicians, having been touched off by the analytic geometry of Descartes and the calculus of Newton and Leibnitz, was focused chiefly upon modern analysis, as well as upon algebra and geometry. As for elementary arithmetic, it was, comparatively speaking, neglected. With the universal acceptance of Hindu-Arabic notation and the recognition of the utility of the decimal notation, the "books were closed" for the time being.

The dual role of arithmetic

Arithmetic from 1850 to the present time may be said to play a dual role in the cultural history of mankind. The more familiar role, and the more prosaic, is that of handmaiden to the arts and sciences, as well as to business. The extraordinary utility of arithmetical computation, as well as of elementary mathematical analysis, has been aptly described by L. Hogben, H. G. Wells, Herbert McKay, and many others. It is a well-known story which need not be reiterated here.

Perhaps the most spectacular development in the field of computation in the last decade or two has been the amazing development of electronic computers. To be sure, both in theory and in practice, these so-called "giant brains" involve far

more mathematics than elementary arithmetic. The story of the development of computing machines is a fascinating one, too long to be told here in detail. But it is a far cry from Babbage's "analytic engine" of 1850 to the now famous Eniac of 1950. Ironically enough, our culture, insofar as science and technology are concerned, is now at a point where progress, certainly in some areas, is no longer possible by individual effort alone, but requires the co-operative efforts of a group of related specialists. Thus in developing a typical I.B.M. machine, a fifty-man team may well be used: twenty mathematicians, twenty engineers, and ten technicians. That there is a leader of the team, a person who co-ordinates the work and directs the project, does not alter the sober fact that no one person can understand all of the theory and intricacies of the machine.

Notably less familiar, at least to the layman, is the second role played by arithmetic in the last hundred years. Yet in many ways it is more subtle and more profound. We refer to the role of arithmetic as catalyst to the comparatively modern examination of the logical foundations of all mathematics, the search for the "structure" of mathematics. This tremendously significant hallmark of twentieth-century mathematics was touched off first by making arithmetic more abstract and generalized, and then by subjecting algebra and analysis to severe "arithmetization." We shall try to make this clear in a few words.

The search for the structure of mathematics

Pythagoras was convinced that all mathematics could be based on the ordinary numbers 1, 2, 3, Mathematicians of the eighteenth and early nineteenth centuries departed drastically from this naïve point of view by successive extensions of the concept of number from the ordinary whole numbers—extensions to the negative integers and zero, to fractions, to irrationals, to real and to com-

plex numbers—concepts that would doubtless have bewildered Pythagoras. But the middle of the nineteenth century was to witness a revolution: mathematics won a freedom of imagination hitherto unknown. It was anticipated by the invention of non-Euclidean geometry by Lobachevski and the abstract approach to algebra initiated by Peacock, Gregory, and De Morgan, all in the 1830's. The essence of their approach was to regard geometry and algebra, each respectively as an abstract hypothetico-deductive system in the manner of Euclid. A dozen years later, when Hamilton rejected the commutative law of multiplication (saying in effect, "Let us see what kind of an algebra or arithmetic we get if we assume that $a \times b$ does not equal $b \times a$ "), the floodgates were opened. From that moment on, mathematicians devoted more and more attention to deliberate generalization and abstraction, exploring the full implications of postulates, and seeking an underlying structure of mathematics.

Contemporary mathematics

The movement gathered momentum. About 1850, Boole expounded his *Laws of Thought*, foreshadowing modern symbolic logic; about 1875, the nature of the real number system was attacked in earnest by Cantor, Dedekind, Weierstrass, and others. In 1899 the die was cast: Hilbert's logical foundations of geometry sounded the keynote for postulational methods. Accordingly, geometric entities and numbers, as such, became pure abstractions, and the really important question for investigation was the nature and structure of the relations between these abstract concepts. Oddly enough, some ten years earlier, Peano had set forth his set of postulates for common arithmetic and deduced from them, by rigorous logic, the entire body of arithmetic based upon the ordinary, or natural, numbers. So the pendulum returned once more to Pythagoras.

The postulational technique thus initi-

ated proved to be the most powerful single influence of twentieth-century mathematics. Contemporary mathematics is to be distinguished from all previous mathematics in two vital respects: (1) the intentional study of abstractness, where the important considerations are not the things related, but the relations themselves; and (2) the relentless examination of the very foundations—the fundamental ideas—upon which the elaborate superstructure of mathematics is based.

The validity of mathematical reasoning cannot be ascribed to the nature of things; it is due to the very nature of thinking. But the average man or woman does not customarily engage in a level of thinking that involves such abstraction and generalization. It is chiefly for this reason that mathematics repels so many people; the subject is too recondite. In this connection we recall the words of the late Professor C. H. Judd,* who reminds us that

... children are not born with a number system as a part of their physical inheritance; they are not endowed at birth with number ideas in any form. The school puts them in contact with a system of number symbols which is one of the most perfect creations of the human mind. In the course of their acquisition of this system, they learn how to think in abstractions with precision. They learn how to use an intellectual device which no single individual, no single generation, could possibly have evolved. In the short span of a few years a child becomes expert in the use of a method of expressing ideas of quantity which cost the race centuries of time and effort to invent and perfect.

Mathematics is a linguistic activity; its ultimate aim is preciseness of communication. Second only to the mother tongue, the language of number is without doubt the greatest symbolic creation of man. And in some ways it is an even more effective agency of communication than the vernacular. In short, mathematics is a great cultural heritage, and although the beginnings have been lost in the mists of time, it is a heritage we should be proud to transmit to the world of tomorrow.

* Charles H. Judd, *Educational Psychology* (New York: Houghton Mifflin, 1939), p. 270.

A case in point

Is it wise to insist upon
correct usage of *numeral* and *number*
at the elementary level?

WAYNE PETERSON *Seattle Public Schools, Seattle, Washington*

Mr. Peterson is presently teaching at Marcus Whitman Junior High School, Seattle, Washington, and has been a member of both the Seattle Center SMSG Seventh- and Ninth-Grade Teams.

When mathematics is taught, wrote George Boehm, "it is presented mainly as a collection of slightly related techniques and manipulations. The profound, yet simple, concepts get little attention. If art appreciation were taught in the same way, it would consist mostly of learning how to chip stone and mix paints."

These simple, yet profound, concepts pervade all of mathematics. Some seem so obvious as to require no special attention, although a lack of early emphasis upon even these "obvious" concepts may result in difficulty at a later stage in the student's mathematical development. A progenitor of such difficulty is often the failure in the beginning years of arithmetic study to make meaningful definitions of terms which must necessarily be employed throughout mathematics, making sure that these definitions are mathematically correct and will not lead to confusion or contradiction at a later date. Embodied within the definitions of such terms are some of the "profound, yet simple, concepts" to which Boehm referred in the introductory quotation taken from his preface to the *New World of Math*.

Definitions and usage

A case in point is the definition of *numeral*, employing as it does the distinction between this concept and the concept of

number. Even though we may recognize this distinction, it is common practice to use the word *number* almost exclusively regardless of which meaning we wish to associate with the word. As I hope to illustrate effectively, this is acceptable practice some of the time, but upon other occasions correct usage of both words, *number* and *numeral*, should be insisted upon.

I was prompted to write this paper after reading the article "Number, Numeral, and Operation," in the May, 1960 issue of *THE ARITHMETIC TEACHER*, wherein the author advised against correct usage of number and numeral at the elementary level on the grounds that it would serve no useful purpose and would create confusion where none had existed. To use an example given in the article, the simple multiplication problem $8 \times 12 = 96$ becomes "the product of the numbers represented by the numerals 12 and 8 is the number represented by the numeral 96." This is of course an absurd insistence upon "correctness." But because it is not useful to make the distinction in this specific instance is no proof of universal inapplicability.

In this article it was also indicated that both the Commission on Mathematics and the School Mathematics Study Group advise "that we should use the one term

'number' for both meanings." Having taught both the seventh- and ninth-grade SMSG material and having read the Commission Report in some detail, I find nothing in either which would lead me to draw this conclusion. In fact, in the SMSG "Mathematics for Junior High School," page 23, it is clearly stated that *numerals are symbols for numbers* and this understanding is indispensable to the successful completion of the work that follows. The Commission Report cautions against insistence upon too much correctness which could result in "pedantic circumlocution," as exemplified by the description of the multiplication example cited above.¹ But nowhere do I find anything beyond a caution to use discrimination in choosing when to employ correct usage and when to allow elliptical liberty in the use of the word *number* for both meanings.

So while I respect the writer's understanding of the problems inherent in teaching mathematics at the elementary-school level and commend his article for your perusal, I cannot agree that the concept of numeral should be avoided in the teaching of arithmetic. The rest of this paper will be devoted to a discussion of *number* and *numeral* and some instances where correct usage will contribute to clarification and understanding.

The child's intuitive idea of number

The concept of *number* is abstract and undoubtedly too sophisticated a notion to be treated formally at an early age.² Nevertheless children do possess an intuitive idea of number. Ask a child what is meant by the number "two" and he will probably hold up a pair of fingers. This intuitive understanding of number is entirely adequate until the youngster reaches a much later stage in his mathematical development. In contrast, the idea of *numeral* is concrete and easily understood at an early age.

A numeral is a symbol which denotes a

number or perhaps more simply, a numeral is the name of a number. Surely, this is simple enough for a child to understand. Numerals are merely the marks we write with chalk or pencil and the words we use to name numbers. The sooner the distinction between number and numeral can be made apparent, the better. Yet, this distinction has been almost universally ignored in the teaching of arithmetic. True, we have Roman numerals, but these are customarily treated as a breed apart and the reason for calling them numerals is obscure.

Different numerals but the same number

Traditionally, we have spoken of "numbers which have the same value" or of "numbers which equal each other." This is a mathematical absurdity. Different numbers cannot be equal and to say that they are breeds confusion. By the introduction of the meta-mathematical term "value" we open the door to a variety of connotations which the individual child may attach to the work. Later when we may wish to attach a precise mathematical meaning to the word value (as in "value of a function") the earlier impression may cloud understanding. Above all, the term "value" is completely unnecessary in this connection and for this, if no other reason, should be avoided.

The alternative is obvious. To say that $1/4$ and $9/36$ are simply different names for the same number (or different numerals which denote the same number) obviates once and for all the necessity to talk about "different numbers which have the same value." How many numerals exist which name this number? An infinite variety of them—names without end! No matter how many names are supplied, someone else can always give just one more.

When decimal fractions are first introduced, it should be explicitly understood that this is just a different way to name familiar numbers. What is meant by 28

per cent? Is this a new kind of number? Again your answer to the child should assure him that he has already met all of the numbers of arithmetic³ and that per cent is just one more way to name these numbers—a way which through custom has become useful within certain contexts.

If a child has a garden containing three rows of corn and two rows of peas, it may be more important to know that there are three of one and two of the other than it is to know that there are five rows altogether. The point here is that $3+2$ is just as good a name for the number of rows in the garden as is 5, and it could very well be the best (or simplest) name for the number in question. Therefore $3+2$ may have one of two meanings attached to it, both of which can be understood at an early age. On the one hand we are directed to add, while on the other we should understand that $3+2$ is just one of the many names for the number five. This prior understanding would help dissipate the urge experienced by most beginning algebra students to combine an expression such as $2x+y$ into a single term.

Making fractions understandable

When fractions are "changed in form" by multiplying both terms of the fraction by the same number (using the multiplication property of one), most children are understandably convinced that the number is undergoing some sort of transformation. They learn to follow the rule to obtain answers, but there is often a complete lack of understanding of the concepts. Yet when they know that it is just the names of the numbers that are transformed, the numbers remaining unchanged, as indeed they must, things begin to make sense, and along with understanding will come increased proficiency and a much greater degree of retention. Adding the numbers three-fourths and five-sixths is not impossible even though we have no prescribed way to add the numerals $3/4$ and $5/6$. Since our supply of numerals that name these numbers is unlimited, let us

choose the ones that will best suit our purpose—in this case, $9/12$ and $10/12$.

Division of fractions by "inverting the divisor and multiplying" is the one rule that perhaps has the least meaning to a child. Some teachers find it advantageous to treat the quotient of two fractions as a single fraction in complex form. This depends upon prior understanding that the quotient of two numbers can be expressed as a fraction. With this understanding, the

$$\text{problem } \frac{6}{7} \div \frac{3}{5} \text{ can be written } \frac{\frac{6}{7}}{\frac{3}{5}}.$$

This complex fraction is then viewed simply as the name for a number, and the job to be done is to find the simplest name for this number. The desired numeral is found as follows.⁴

$$\begin{aligned} \frac{\frac{6}{7}}{\frac{3}{5}} &= \frac{6}{7} \times \frac{5}{3} = \frac{6 \times 5}{7 \times 3} = \frac{30}{21} = \frac{10}{7} \end{aligned}$$

With the understanding that $\frac{3}{3}$ names the number one and therefore multiplying by $\frac{3}{3}$ changes the numeral but not the number, this method for determining the quotient of two fractions becomes meaningful. Since the student would not be expected to show each step in detail as was done here, it is computationally simple as well as mathematically sound.

Number lines and numerals

When maturity permits a greater degree of abstraction, it is useful to introduce the "number line" wherein the numbers of arithmetic are each associated with a point on the line. Geometrically, a line is a set of points infinite in number. The numbers of arithmetic are also infinite in number. Between any two points on the

line there exists another point. Similarly, between any two numbers of arithmetic there exists another number of arithmetic (we can always find such a number by averaging the two given numbers). We can therefore establish a correspondence between the points on the line and the numbers of arithmetic.

To construct a number line, mark a zero point on a straight line and establish a unit of length that will allow you to mark off the points corresponding to the whole numbers. The remaining numbers of arithmetic will then correspond to points between those so marked. In this way, each number corresponds to a unique point on the line, but a specific point may be designated by an infinite variety of numerals, each of which names the same number.

One important use of the number line is to aid in determining the order or relative size of numbers—a job often complicated by the use of different numeral forms. To a child, 90 per cent looks bigger than 36. But in the act of attaching these labels to points on the line, understanding is forced on the child.

Some teachers have developed “number base” units of work for superior students. I am not advocating this, for my personal belief is that this type of material can be introduced before the youngster has achieved sufficient mathematical maturity to appreciate the beauty of this type of thing (the SMSG material includes a unit on numeration at the seventh-grade level), but it may be desirable to discuss briefly “numeral” within this context.

Our base ten notation dictates the value due to the position of a digit as well as the value due to the specific digit used. Consider the numeral 42. This numeral is understood to mean “4 tens and 2 ones.” If this same numeral is considered within the context of base five, the value of the “4” due to position is no longer “4 tens.” The numeral 42 must now be understood to mean “4 fives and 2 ones.” Similarly, in base twelve, the numeral 42 means “4 twelves and 2 ones.” While the structure

of these various numeration schemes is identical, we see that a single numeral may represent as many different numbers as there are bases under consideration. As interesting as this may be, I do not believe it advisable to burden your students with this information unless you intend to do a fairly thorough job of teaching a unit on number base. A little dab of information at the wrong time might take the fun out of exploring a new dimension of mathematics at a later date.

In your day-to-day teaching, other instances will undoubtedly occur wherein an understanding of the number-numeral relationship will be of help in clarifying troublesome concepts for your students. If used with discrimination, the observance of this distinction will serve you as an extremely useful pedagogical tool.

Notes

1. See page 4, footnote, “Appendices,” Report of the Commission on Mathematics.

2. See the opening paragraphs of “Number, Numeral, and Operation,” by John H. Clark, *THE ARITHMETIC TEACHER*, VII (May, 1960) 222.

3. Here we define the numbers of arithmetic as the set of nonnegative rational numbers. These may be thought of as the set of positive fractions and zero. The whole numbers are included in this set, since they may be written in fraction form with a unit denominator.

4. An equally acceptable method is to multiply numerator and denominator by the reciprocal of the denominator (this is the rationale for the “invert and multiply” rule).

$$\frac{\frac{6}{7}}{\frac{3}{5}} = \frac{\frac{6}{7}}{\frac{3}{5}} \times \frac{\frac{5}{3}}{\frac{5}{3}} = \frac{\frac{6}{7} \times \frac{5}{3}}{\frac{3}{5} \times \frac{5}{3}} = \frac{\frac{30}{21}}{1} = \frac{30}{21}$$

$$= \frac{10}{7}$$

Highlights of a summer conference

CATHERINE LINN DAVIS

University of Missouri, Columbia, Missouri

Mrs. Davis is an instructor in the College of Education and supervisor of grade two in the University of Missouri Laboratory School.

Across the nation today teachers and students are being offered an opportunity for professional inspiration and growth under conditions unique to our times. Under the auspices of the National Science Foundation, an increasing number of universities and colleges are conducting institutes and conferences in science and mathematics. Participants in these institutes receive stipends that make it financially possible, and often profitable, for them to attend. In many instances, they receive college credit toward advanced degrees for the course work involved.

During the past summer I had the good fortune to participate in the Summer Conference in Arithmetic and Mathematics for College Teachers of Arithmetic Curriculum and Methods and Supervisors of Elementary Arithmetic Programs held at the University of California, Berkeley, California, from July 31 through August 12. The conference was sponsored by Sacramento State College and the University of California through a National Science Foundation grant.

Conference goals and routines are set up

The purposes of the Berkeley Conference were to evaluate the teaching of arithmetic in the elementary school and to review critically the preparation of elementary teachers in the light of the demands of our rapidly evolving, techno-

logical society and of the teacher's changing role.

Fifty-eight participants attended the conference, coming from twenty-three states and representing fifty-two different college and university departments of mathematics and education, as well as supervisory staffs of elementary education in public school districts.

Exclusive use of one of the dormitories on the campus of the University of California for housing, meals, and conference rooms provided the atmosphere and opportunity for a friendly, natural, free discussion of professional ideas and problems which would not have been possible otherwise.

Well-known speakers provide stimuli to thought

Each morning's work consisted of lectures by two outstanding specialists in the fields of mathematics and mathematics education. The afternoons were devoted to a question period for each of the morning's speakers and to a two-hour group discussion in one of three areas of special interest: college content and method, in-service training programs, and experimental programs.

The conference speakers were Dr. Howard F. Fehr, professor of mathematics, Columbia University; Dr. Izaak Wirszup, associate professor of mathematics, University of Chicago; Mildred B. Cole, vice-president of The National Council of

Teachers of Mathematics; Dr. E. G. Begle, professor of mathematics, Yale University; Dr. H. Stewart Moredock, professor of mathematics, Sacramento State College; Dr. Warwick W. Sawyer, Wesleyan University; Dr. Norman Rudy, professor of statistics and engineering, Sacramento State College; Dr. Roy Dubisch, professor of mathematics, Fresno State College; Dr. Ivan Niven, professor of mathematics, University of Oregon; Dr. May H. Maria, associate professor of mathematics, Brooklyn College; Dr. Maurice R. Ahrens, professor of education, University of Florida; Dr. Carl B. Allendoerfer, University of Washington, president of the Mathematics Association of America; Dr. Francis J. Mueller, assistant to the president, University of Rhode Island.

Areas of agreement

In my opinion, these speakers were in complete agreement on the need for, and the general direction of, change in the area of the elementary curriculum and of teacher-training programs. They appeared to agree that while we cannot anticipate with assurance the future mathematical requirements of our students, if mathematics is to serve as an effective tool for solving the problems of this age, we must give major emphasis to the development of an understanding of the fundamental concepts of modern mathematics. They stressed that the negative attitude of many teachers toward mathematics must be altered. Teachers must understand and enjoy mathematics if they are to inspire pupils to do so. Mere computational skill is inadequate. Teachers must know the rationale. They must have flexibility within the computational structure. They must be able to relate the psychological principles of learning to the teaching of arithmetic. Certain modifications and improvements must be made in the mathematics curriculum at all levels if this is to be accomplished. The speakers emphasized a need for in-service training programs. They indicated approval of the current

trends to alter the traditional elementary curriculum to include: emphasis on the building of concepts, greater use of the discovery method of instruction, greater mathematical correctness, preciseness of language, complete rationalization of algorithms, and earlier introduction of many topics, i.e., algebra introduced earlier and taught over a longer period of time, more intuitive geometry placed lower in the grades.

The participants at the conference investigated the following experimental programs currently being conducted: the Madison Project; Schott Method; Cuisenaire-Gattegno Method, School Mathematics Study Group, grades 4, 5, and 6; Geometry in the First Grade, Stanford University; the Scott, Foresman Experimental Mathematics Project, grade 7; University of Illinois Arithmetic Project; University of Maryland Mathematics Project; Greater Cleveland Mathematics Program materials. Their investigation of these programs revealed that virtually all experimental programs in progress today and a number of the new text materials will require additional understanding or preparation on the part of teachers. Future pre-service and in-service programs must provide this background.

In addition to the investigation of these experimental programs, the conference personnel considered the current programs of their own institutions. In the light of the conference speakers' opinions and of their own deliberations, the group made the following recommendations concerning the mathematics curriculum, both content and methodology courses, which such institutions should require for graduation and certification of elementary teachers.

Conference recommendations on content

1. One year of algebra and one year of geometry on the high school level. These courses should not be a prerequisite for entrance into the teacher-training program but should be prereq-

quisite for admittance to the mathematics content course for teachers.

2. A sequence of approximately nine semester-hours for content and methods courses in mathematics with two-thirds being devoted to content courses and one-third to a course in the methods of teaching arithmetic to elementary-school children. It was suggested that these courses be taught at upper division levels following the courses in child growth and development and in the psychology of education.
3. That the content portion of this sequence include:

First course: Elementary Number Concept
(3 to 4 semester-hour credits)

Introduction to sets (elementary concepts)

Sets and subsets

Inclusion, relation

Union, intersection, complement

Concept of number

Cardinal and ordinal

Equals and ordering relations

Systems of numeration (caution: a limited discussion of early systems)

Suggested systems and principles:

Egyptian—base ten, additive

Roman—modified base ten, additive

Mayan—base twenty, additive, consistent use of symbol for zero

Chinese-Japanese—symbols for one through nine, and symbols for 10, 100, 1000, multiplicative

Babylonian—base sixty, remnant in seconds and minutes

Hindu-Arabic—base ten, positional value, zero

Other bases—principles of positional notation of Hindu-Arabic, and zero, e.g., in a binary, octal, or duodecimal system

Modular systems

Mod. prime (e.g., 5 or 7)

Mod. composite (e.g., 6, 8, 12)

Addition and multiplication tables

Closure, commutative, associative and distributive principles

Additive and multiplicative identities

Additive and multiplicative inverses

Solution of simple equations

Proper divisors of zero

The number system

Natural numbers (counting numbers)

Definition of addition and multiplication

Closure, commutative, associative and distributive principles

Prime and composite numbers; l.c.m., g.c.d.

Zero and its properties

Solution of simple linear equations

Subtraction defined as the inverse of addition

Integers (natural numbers, their negatives, and zero)

Definition of addition, subtraction, and multiplication

Closure, commutative, associative, and distributive principles

One and its properties

Solution of simple linear equations

Division defined as the inverse of multiplication

Rational numbers

Definition of addition, subtraction, multiplication, and division

Closure, commutative, associative, distributive principles

Solution of simple equations

Terminating and periodic decimals as rational numbers

Irrational numbers

π (pi), nonrepeating, nonterminating decimals

Real numbers

Rationals and irrationals

Number line

Solution of simple equations

Complex numbers

(Introduction and elementary properties)

Measurement

Concept of measurement

Number as a quantitative

Units of measurement

Precision and accuracy

Approximate computation

Ratio, proportion, percentage

Second course: Logic and Geometry
(3 to 4 semester-hour credits)

Elementary logic

The language of logic and sets
Statements as propositions
Connectives (and, or) and negation (not)
Implication (if, . . . then), syllogism and direct proof
Logical reasoning
Equivalent statements to lead to indirect proof
Necessary and sufficient conditions (if and only if)
Sets and Venn diagrams

Plausible reasoning

Estimation and guessing
Inductive reasoning

Informal geometry

Use of ruler, protractor, and compass
Scale drawing, perspective, indirect measurement
Definitions and properties of some simple figures, such as triangle, circle, square, rectangle, etc.
Intersection and parallelism of lines
Similar triangles
Three-dimensional concepts, cube, sphere, pyramid, cylinder, cone, regular solids
Perimeters, areas, volumes
Maps, latitude and longitude, great circles

Co-ordinate geometry

Rectangular co-ordinate system
Proof in analytic geometry
Graphing a straight line, circle, and other conics
Graphs of inequalities
Graph solution of systems of equations and inequalities

Geometry as a mathematical system

Euclidean geometry
Parallel postulate and its significance
Non-Euclidean geometries (Descriptive introduction)

A greater knowledge and understanding of mathematics than would be obtained from the courses outlined above is

necessary to teach mathematics at the seventh- and eighth-grade levels.

**Conference recommendations
on in-service training**

Since it is necessary to acquaint many teachers and administrators with the need for improvement in the elementary-mathematics program, the group recommended that a strong in-service education program be developed to include:

1. Continuation and/or initiation of conferences and institutes organized on a local and national basis for the purpose of reviewing current experimental programs, discovering ways to implement recommended changes in local mathematics curricula, and providing effective in-service training related to content as well as method.
2. A television course, similar to "Continental Classroom," to provide desirable subject-matter background for elementary teachers.
3. Development of a series of films to help educate teachers in mathematics and its application in the classroom. These should be readily available on a rental basis.
4. In-service education programs at the local district level through utilization of college-sponsored course work, participation in NDEA projects, regional conferences and workshops, district-wide curriculum study, teacher observation of master teachers, a professional library of arithmetic publications, assistance for teachers through district or county consultants, utilization of local television facilities.

It was recommended that local districts select effective leaders as representatives to national institutes and conferences. These representatives could then serve to co-ordinate local elementary-curriculum improvement.

The consensus of the conference participants was that "the key to the improvement of the elementary-arithmetic curriculum and to the greater effectiveness of the elementary teacher lies in the inclusion of increased mathematical content in the curriculum of teacher education both at the pre-service and in-service levels. This should lead to the alteration of the arithmetic program in the elementary schools in the direction of greater mathematical understanding developed through a discovery approach so that it may be more universally applicable both to life situations and to further mathematical study. More effective presentation of mathematics to children is predicated on the assumption that teachers are well versed in the most efficient instructional procedures."*

The personal factor

While these findings are, I believe, of real significance and worthy of imple-

mentation, there were other outcomes for the participants that may well be of even greater immediate importance. Through the impact of the various personalities and the opportunity for the exchange of views, there was generated in the participants not only a wider knowledge of mathematics and methods of teaching arithmetic to elementary children but an attitude of enthusiasm for the task ahead. An increased sense of personal worth was created by the realization that one can make a more adequate contribution toward the goals of our profession. Both the future students of the participant and the employing institution will profit as a result of a more confident, enthusiastic, and able teacher.

I believe that the benefits to the individuals involved, to the institutions by which they are employed, and to their future students are potentially so great that every effort should be directed toward increasing the number of such institutes held and toward encouraging more teachers to seek the opportunities they offer.

* Preliminary publicity release prepared by committee of conference participants.

Call for candidates

The Committee on Nominations and Elections is making plans for the 1962 election. Officers to be elected are: president, vice-president for senior high school, vice-president for elementary school, and three directors.

The Committee invites your suggestions for candidates. Please submit names of able individuals, along with full information about their qualifications, to the Chairman of the Committee on Nominations and Elections, Mr. Eugene Ferguson, Newton High School, Newtonville 60, Massachusetts.

Problems without numbers

WALTER L. KLAS

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Dr. Klas is principal of Woodrow Wilson School.

The present concern to secure challenging programs for more able students has resulted in a number of interesting proposals. It appears that the acceleration versus enrichment problem is not yet settled. Indeed, the proposals for challenging the more able students appear about evenly divided between the alternatives. The important fact is that people are thinking about the problem and many good solutions are being offered and tested throughout the country.

One of the outcomes of this concern has been the interest in the proposals of specific techniques which facilitate the type of arithmetic program which more able students find stimulating. One such technique is offered here. It is hoped that it will be added to the stock of existing techniques which teachers have found profitable.

Getting the pupil to ask, "What facts do I need?"

The proposal is simply that problem situations be given students *without* including the numbers commonly used. The pupils' job in solving the problems thus becomes one of determining *what facts* are needed as well as applying proper mathematical processes to the solutions. This technique can be effective at all levels of the curriculum and can be as complex or as simple as the teacher deems desirable for the group taught.

Let us take an example at a primary level: "Jimmy has a number of marbles.

He is going to give some to his friend Bill and will still have some left. How many will he have left?" To solve this problem, the pupil will need to ask for certain information. This can be done either orally or in writing. He will need to know how many marbles Jimmy had in the beginning and how many he gave Bill. Knowing these two facts, the pupil then can easily apply subtraction to solve the problem. The important part of the solution is that the pupil considered what facts were needed not what mathematical process to apply.

Providing for growth in ability

At a higher level the problem situation can be changed to: "How much will a vacation cost a family?" This type of problem can be made quite complex depending upon the information given the pupils as they work through the problem. Such items might be considered as size of family, length of vacation, method of travel, manner of eating, cost of gasoline, oil, and other car expenses, and destination. The possibilities of such a problem are virtually endless. In addition, much will be learned by the pupils because the teacher gives only the information asked for and does not protect pupils from making mistakes. All this will contribute to the thinking which must precede the application of mathematical processes.

Among other types of situations which lead to this type of problem are: "How

much will this strip of sidewalk or road cost?" "How much will it cost to paint a house?" "How much will it cost to plow a field?" "What will be the cost of constructing a swimming pool?" These examples will lead readers to other problems involving similar situations. In fact, as teachers work through such problems other variations become evident and can be shared from teacher to teacher and adapted to suit the class involved. Then, too, pupils will offer other situations that lend themselves to the same type of solution.

More able students will be challenged

by such problems because they involve more than the simple application of an already known process in arithmetic. Teachers know only too well the ability of bright pupils to make use of clue words in the solution of problems and thus to do very little thinking in the solution. The type of problem mentioned here lends itself particularly well to individual differences because the information given can be varied to suit individual pupils or given in units other than those requested by the pupil. Thus, the pupil would find a need for conversion to other units in the solution of his problem.

Will you contribute to a forthcoming yearbook?

The Board of Directors of The National Council of Teachers of Mathematics at the April, 1960 meeting approved the preparation and publication of a yearbook to be devoted to the problem of the mathematical education of the talented student in grades K-12.

It is intended that the yearbook be a *source book* of topics and materials which have been found useful in enriching the mathematics program of talented students, but which are not parts of either traditional or experimental courses.

Under the chairmanship of Julius H. Hlavaty, the editorial committee, some members of which are listed below, has accepted the responsibility of preparing the grade-level sections as indicated here. If you have material that you believe would make a contribution toward achieving the purpose of the yearbook, won't you send your contribution to the appropriate member of the committee or to the chairman?

Please submit your contribution no later than February, 1961. Where grade levels listed here overlap, send the ma-

terial to either committee member, but not to both.

K-8	VINCENT J. GLENNON Director, Arithmetic Center Syracuse University Syracuse 10, New York
7-10	JOSEPH L. PAYNE 3019 University School University of Michigan Ann Arbor, Michigan
9-11	HENRY SYER Kent School Kent, Connecticut
12, Honors	HARRY D. RUDERMAN Hunter College High School 930 Lexington Avenue New York 31, New York

Full credit will be given to each contributor whose material is used.

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Testing—without tests

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Mr. Harrison is a junior high school mathematics instructor.

We often observe that students do poorly on tests in mathematics because of the psychological element of fear. This element may be reduced to a minimum with the same amount (possibly more, in some instances) of learning, and the teacher still achieves the primary purpose (often forgotten by teachers themselves) of a test.

Instead of the usual Friday test with mathematical manipulation, in its place substitute the writing of an "experience" book, thus involving the pupils with the subject instead of setting them against it. This experiment was carried out in a junior high school class of forty students: thirty-two girls and eight boys. It was interesting to note that for three report periods, or eighteen weeks, there was 100 per cent participation on the part of the students.

While the student was given instruction as to content required by the instructor, he was also asked to use his imagination in the construction of his book. Artistic ability and imagination were to be used by the student with "no strings attached" by the instructor. Also, should the student wish to add any historical comment or delve deeper into the subject, he was encouraged to do so. A final restriction was most important: the student was not to repeat any mathematical illustration used in the class discussion.

Needless to say, the teacher worked as diligently as the students. Making corrections (of English as well as of mathematics) and comments on each paper consumed approximately eighteen hours. Books were not handed back until each

had been read carefully.

A chart of errors enlightened the teacher as to the points that would have to be strengthened by reteaching, in both group and individual instruction. All errors were to be corrected, and each week's "experience" was to be combined in a final form. At the end of the semester, the booklet was then assigned a grade.

If you look at the textbooks and the workbooks, it is obvious that many authors believe manipulation is all that is necessary. Explanations of processes are mechanical, answering the question "How?" rather than the question "Why?" The student is left with little to think about. Another disturbing element is the lack of continuity. The problems are often isolated, and there is little or no exercise in sustained thinking.

The "experience" booklets give the student practice in sustained thinking; furthermore, the student must learn to express ideas so that others can understand them. The minimal benefit of this method—so designated because not strictly within the limits of mathematics—is that the student learns to listen attentively, to take notes regularly, and to shape a continuous product as a reward for doing the daily prerequisites.

The maximal benefit derived is the appreciation of the struggle the human mind makes to cope with the quantification of experience. There is, too, the satisfaction of trying to understand what makes a process work. Equally important is the fact that you have students who are enchanted with mathematics!

The use of models in the teaching of mathematics

ROGER OSBORN *University of Texas, Austin, Texas*

Professor Osborn is a member of the Department of Mathematics, University of Texas

Instructional materials make up an important part of the equipment of the effective teacher of elementary- or secondary-school mathematics. The distinction between number and numeral is being made with increasing consistency and emphasis in programs of mathematics instruction being currently evolved. The development of this concept may, for many teachers, point up a need for new evaluation of the role of instructional materials in the classroom. The importance of the fact that the name of a thing and the thing itself are not the same has become more and more evident in mathematics programs as teachers consider numerals as symbols we use to denote numbers.

Those aspects of physical situations which involve number are those which concern the mathematician, the teacher of elementary mathematics, and the child learning mathematics. But only rarely, if at all, do mathematicians, teachers, or students work with the number aspects of the actual physical situation. Rather, they work with the number aspects of models of the actual physical situation. The following problem illustrates the point.

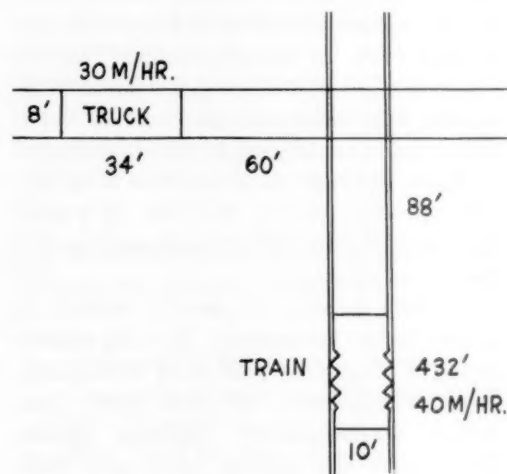
Diagrams as models

A truck 8 feet wide and 34 feet long is traveling down a straight street at 30 miles per hour. It is approaching a railroad track which crosses the street at right angles. A train is also approaching the crossing. The train is 10 feet wide, 432 feet long, and is approaching the cross-

ing at 40 miles per hour. At a given instant the front of the truck is 60 feet from the crossing, and the front of the train is 88 feet from the crossing. Will there be a collision? If so, will the train strike the truck or the truck strike the train?

How do we go about solving such a problem? Do we physically recreate the situation? We could, but in so doing we would be constructing a model of the original situation. We might draw a diagram—a model of the situation. Even the spoken or written words are models of the situation since the symbol 8 feet certainly is not the actual width of the truck. Even if we were the actual observer of the physical situation, we would doubtless form some mental model (mental image) to solve the problem.

Suppose we diagram the problem—this being the most likely treatment.



The importance of accuracy

We see that a drawing made nearly to scale may be most helpful in our visualization—and for teaching children I really believe we want our models to look as much like the actual physical situation as is feasible so that no misconceptions may arise from improper visualization—but only infrequently is actual scale drawing necessary.

From our visualization of the situation, probably best obtained from the model, we see that the truck must go $60 + 10 + 34$ feet to clear the entire intersection. This is 104 feet. Since the train is moving faster than the truck, it will cover the 88 feet between its initial position and the intersection before the truck (which is moving more slowly) can clear the intersection. We know already, then, that there will be a collision.

There is something additional to be learned from this model. It is that the model must represent the situation faithfully enough to allow valid conclusions to be drawn. Suppose the model is drawn using only a spot for the intersection and a spot for the truck and a spot for the train, and chalk or pencil lines for the tracks and road. In such an event, one might conclude (erroneously) that the truck could cover the distance to the intersection in $1\frac{4}{11}$ seconds while the train would take $1\frac{1}{2}$ seconds, and hence that there would be no collision.

Of course, if the train had been 500 feet from the intersection, then the point or spot diagram would have been a sufficiently faithful representation to allow the problem to be worked.

Progressing in understanding from models to symbols

Consider another example. If in the classroom one wishes to show how 8 horses and 14 horses may be “added,” one (in the absence of horses) may represent the situation by pictures, tongue depressors in a place value chart, chalk marks on the

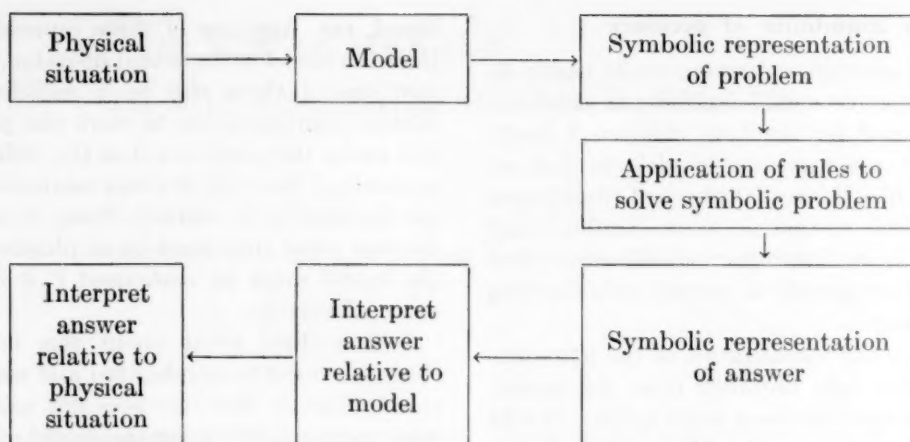
board, etc. Any one of these representations is a model of the actual situation, and any one of them will be a sufficiently faithful representation to work the problem under the condition that the children understand that this is a representation of the problem to be worked. Here, then, in another point that must be emphasized—the model must be *understood* if it is to serve effectively.

Still a third point about this model “theory” must be emphasized and understood. That is that one does not usually work mathematics using the model of the physical situation, but the mathematics is done using a still more sophisticated model—a model of the model. What is this more sophisticated model? It is a set of symbols which we use—numerals; plus, minus, times, and division signs; equals signs; letters, and so on—in working the mathematical problem. In the problem of the truck and train, we do not compute times and distances by using the diagram, but we write down symbols expressing relations between distances, rates, and times, and we manipulate these symbols according to certain laws or rules. From these manipulations we obtain certain results.

Now, what do we do with these symbolic results? We relate them, however briefly, to the model, and then in turn we relate this to the physical situation. This gives us an over-all picture of the problem-solving process using the model theory. We start from an observed physical situation, or some concept of it, go to a model, go from there to a symbolic representation (a more sophisticated model) of the model, solve the problem in symbols, and then retrace the route from symbolic result to model to physical situation, drawing all the appropriate conclusions at each step of the return trip.

Appropriate teaching aids

Having examined this approach to problem solving, consider teaching aids as they relate to our model theory and as the



theory relates to them. Most of the teaching aids used by teachers are either themselves models or allow you to construct models of the problems you actually want to solve. As an example, the abacus allows you to make a model of a problem in which addition or subtraction is to be performed. The place value chart may be considered to represent more clearly the problem since one unit is not necessarily used to represent ten units. The Monroe Educator, a cash register, a speedometer, or an adding machine makes it possible to construct a model in terms of numerals on a

dial instead of beads on a rod. With a chalkboard and piece of chalk many kinds of models may be constructed.

An understanding of the use of models in teaching and learning mathematics enables the teacher to help students comprehend more fully the operations they perform in solving problems. Students have problems that they can most readily resolve when they are able to distinguish between the actual situations about which they desire information and the model situations they utilize in finding this desired information.

Ward Edwards of the University of Michigan knows of a designer who says: "Man is the only small, general purpose computer capable of complex control which is mass-produced free by unskilled labor." Edwards was quoting this gifted phrasemaker as an example of machine-centered philosophy. Edwards insists men excel machines, and it is incorrect to start by asking what functions machines can perform and then let man do what is left.

—*The American Psychologist*, August, 1960

Percentage— noun or adjective?

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Problems dealing with per cent have usually been considered among the difficult topics in the arithmetic curriculum in the elementary school. Adults as well as youngsters will readily confess to their lack of understanding of per cent. The same is true of students in refresher courses in mathematics who list the topic of per cent as one of the least-understood topics in arithmetic.

The difficulties in teaching per cent have been described very effectively by Kinney in a recent article in *The Mathematics Teacher*.^{*} One of the misunderstandings associated with per cent that was not mentioned by Kinney, and is not mentioned by others, is a semantics or vocabulary difficulty. The term "percentage" is used in two ways, as a noun and as an adjective. These two concepts have been interchanged by scholars and research students as well as by the general public, and this has often resulted in conclusions that are not warranted by the facts.

All per cent problems deal with three factors: the number being compared, the number to which it is compared, and the ratio expressed as hundredths. The number being compared is the percentage, the number to which it is compared is the base, and the ratio expressed in hundredths is the per cent. The relationship between these three factors is expressed by the well-known formula, $p/b=r$.

^{*} Lucien B. Kinney, "Teaching Percentage for Understanding and Use," *The Mathematics Teacher* 51 (January, 1958), 38-41.

In the example $\frac{40}{50}=.80$ or 80%, 40 is the percentage, 50 is the base, and .80 is the ratio or per cent. The phrase "per cent" refers to the ratio. The word "percentage" should refer only to the number being compared. It is, however, used as an adjective to describe problems containing per cents. The expression "percentage problems" is familiar. But when one says that the percentages increased because the per cents increased from 7 per cent to 8 per cent, the statement may, or may not, be true. It is quite possible for the per cent to increase although the percentage decreases, and also for the per cent to decrease while the percentage increases.

If a teacher has a class of 40 pupils and 30 of them receive a passing grade, he can say that 75 per cent of the class received passing grades. If his class the following semester contains 50 pupils and 35 receive passing grades, only 70 per cent of the class received passing grades. The percentage increased from 30 to 35, but the per cent of pupils who passed decreased from 75 per cent to 70 per cent.

The use of the word "percentage" when "per cent" is really meant can often be misleading. Three examples taken from a recent research bulletin illustrate the confusion of words.

[Example 1] Large high schools (1000 or more students) edge out small (under 300 students) and medium-sized schools (300-999 students) of this type with over 57 per cent enrolled in mathematics courses. Differences in percentage enrollments among the three sizes of these schools are slight, however, and the over-all figure is 56.5 per cent.

The second sentence in the first example is obviously incorrect. The differences in percentage enrollments are great, for the larger-sized schools have a larger base. What was meant was that the differences in per cent enrollments were small.

[Example 2] But markedly in mathematics the percentage of graduates with *four* or more years moves up with school size—small, 9.7 per cent; medium, 15.3 per cent; and large, 17.3 per cent.

Here in the second example it is true that the percentages as well as the per cents moved up. Since only the per cents were given, the author could just as well have said that the per cent moves up with school size.

[Example 3] Although a very high percentage of these teachers believe that learning how to use various library resources should be a part of the education of all boys and girls . . .

The third example was accompanied by a chart that made the statement itself misleading. The chart showed that almost 60 per cent of the music teachers consider library materials important, but only 20 per cent of the mathematics teachers con-

sider library materials important. Whereas the mathematics teachers show a smaller per cent, the percentage for mathematics teachers is probably much higher than for music teachers because the number of mathematics teachers is much larger than the number of music teachers.

If the phrase per cent, rather than percentage, were used as both noun and adjective, no confusion would result. If the phrase per cents were substituted for percentages in the above quotations, there would have been no change in the meanings of the statements. Mathematics teachers could help overcome this confusion in the topic of per cent if they would use the word percentage when the actual number is given, and reserve the phrase per cent for the ratios only. Let us talk about per cent problems rather than percentage problems. Let us say that the per cents increased rather than the percentages increased, unless we really mean that the actual numbers increased. There are enough difficulties in teaching per cent without adding the semantic or vocabulary difficulty.

Notice of Annual Business Meeting

The Annual Business Meeting will be held at the Conrad Hilton Hotel in Chicago on April 6, 1961. This meeting will be one of special significance to the members of the Council, for at this time proposed amendments to both the Bylaws and the Articles of Incorporation will be presented.

For some time the Board of Directors has felt that the objectives of the Council could more readily be realized if certain changes were made in the Bylaws. In the fall of 1959, a Bylaws Revision Committee was appointed. As the result of much study and deliberation by both the committee and the Board of Directors, certain important revisions have been prepared for the consideration of the members.

It is important for several reasons that a non-profit organization such as ours have a tax-exempt status under the Internal Revenue Code.

The Council, since it became a department of the National Education Association, has been able to take advantage of the latter's tax exemption. However, it is questionable whether this practice could be adequately defended before the courts if it were challenged. So that the Council may apply for its own independent tax exemption, certain amendments of the Articles of Incorporation are required. These amendments have been worked out in consultation with an attorney and have been approved by the Board of Directors.

A statement of the proposed amendments both of the Bylaws and the Articles of Incorporation will be mailed from the Central Office to each member of the Council on February 27. It is hoped that every member will study these materials thoughtfully and that a large group of members will be present to discuss and make decisions on them at the Annual Business Meeting on April 6.

M. H. AHRENDT, Executive Secretary

Grouping—an aid in learning multiplication and division facts

EDWINA DEANS

Arlington Public Schools, Arlington, Virginia

As elementary-school children progress in their study of mathematics, they deal with numbers which are increasingly more complex and abstract. Around the third or fourth year in the grades, children's work with operations requires an understanding of numbers of two or more places and a knowledge of regrouping in terms of our basic unit of ten. Attention to grouping arrangements assists children in their discovery of multiplication and division facts, and it assists them in the use of these facts in the operations of multiplication and division.

The end goal of memorization of facts for immediate recall is facilitated if chil-

dren are first provided with opportunities to explore and discover facts for themselves and to see relationships among them. Toward reaching this goal, you might try some of the following activities with grouping arrangements, adapting them to the learnings which are appropriate for your own class at this time.

Suggested activities using grouping in multiplication

1. Find equal groups for selected numbers along a number line like that shown in Figure 1.

Use the number line (Fig. 2) to illustrate the commutative law which states

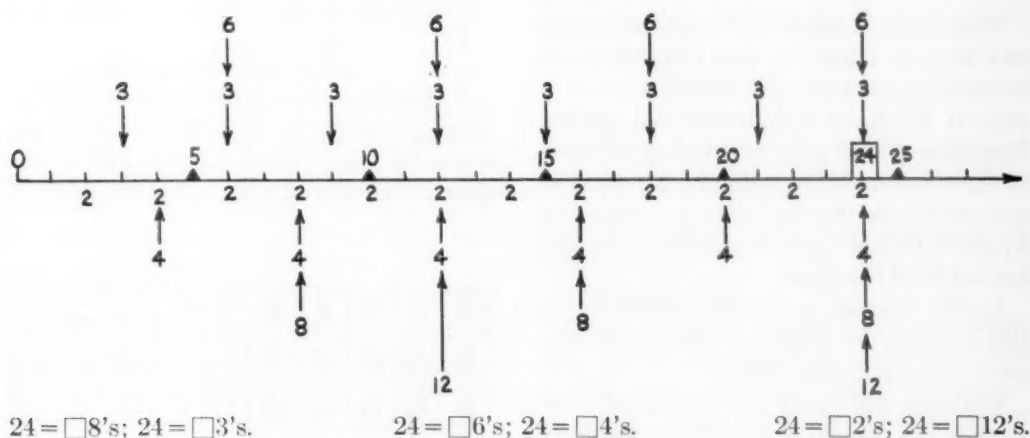


Figure 1

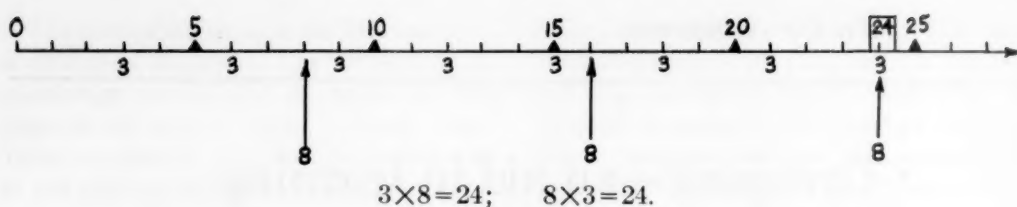


Figure 2

that the product is the same regardless of the order of the factors. Three groups of 8 = 24; eight groups of 3 = 24.

2. Locate groups along the rows of ten on a 100-dot chart.

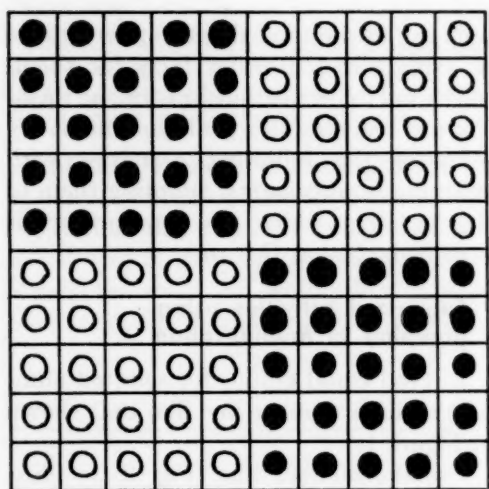


Figure 3

Provide each child with a 100-dot chart like that in Figure 3. The chart may be duplicated or made by children from a 10x10 block of 1/2-inch squared paper. Five dots of one color and five of another on each row make grouping by 5's possible and eliminate the necessity for counting by ones. Colors may be alternated on the lower half of the chart.

Guide children in locating groups along the rows of ten. Figure 4 illustrates the procedure for groups of six.

Children will apply the associative law as they use the chart to regroup numbers for ease in finding answers.

Two 6's = $6 + (4 + 2) = 10 + 2$.
 Four 6's = $18 + (2 + 4) = 20 + 4$.
 Seven 6's = $36 + (4 + 2) = 40 + 2$.
 One 6 = $5 + 1$ or 6.
 Two 6's = $10 + 2$ or 12.
 Three 6's = $12 + 6$ or $10 + 8$ or 18.
 Four 6's = $18 + 6$ or $20 + 4$ or 24.
 Five 6's = $24 + 6$ or 30.

Other facts for 6 may be discovered in a similar manner.

Guide children in using the same type of chart to assist them in completing exercises set up in such a way that the relationships among facts will be stressed.

- { Two 6's =
- { Four 6's =
- { Five 6's =
- { Six 6's =
- { Four 6's =
- { Eight 6's =
- { $30 = \square 6$'s
- { $42 = \square 6$'s
- { $60 = \square 6$'s
- { $54 = \square 6$'s

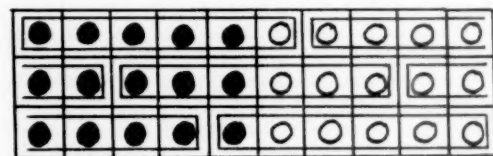


Figure 4

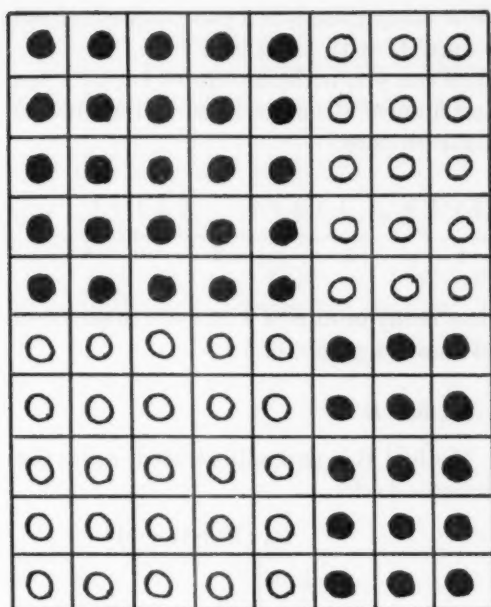


Figure 5

8's (Fig. 5). Methods include using groups of five and ten, using higher decade addition and subtraction together with multiplication facts, and doubling known multiplication facts.

Two 8's = $8+8$ or $(2 \times 5) + (2 \times 3)$.

Four 8's = $16+16$ or $(4 \times 5) + (4 \times 3)$.

Three 8's = $16+8$ or $(3 \times 5) + (3 \times 3)$.

Six 8's = $24+24$ or $(6 \times 5) + (6 \times 3)$; or $(3 \times 10) + (6 \times 3)$; or $(5 \times 8) + (1 \times 8)$.

Seven 8's = $(5 \times 8) + (2 \times 8)$; or $(7 \times 5) + (7 \times 3)$.

Nine 8's = $(10 \times 8) - 8$; or $(5 \times 8) + (4 \times 8)$.

Long division as regrouping

This section describes two ways by which children may determine how a larger group, such as 365, may be rearranged into equal groups of 5. In Method A, tens and ones, or hundreds, tens, and ones are regrouped separately. Any remainder of hundreds must be regrouped with tens; any remainder of tens must be regrouped with ones. In Method B, groups of hundreds, then tens, then ones of the divisor are pulled out until the dividend is completely regrouped.

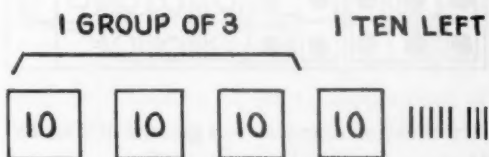
Perhaps the method chosen is not so important as the understanding children have of the method they use. Both methods can be taught with meaning.

See if the series of examples, diagrams, and the explanations given will help your children understand better the method of long division you are teaching. Your talented children in mathematics may find it challenging to learn how to handle both ways.

Method A

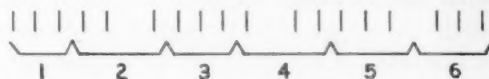
Method A is illustrated with the example $3 \overline{)48}$.

$$\begin{array}{r} 16 \\ 3 \overline{)48} \\ \underline{30} \quad (3 \text{ tens}) \\ 18 \quad (1 \text{ ten} + 8 \text{ ones or } 18 \text{ ones}) \\ \underline{18} \end{array}$$



How many groups of 3 tens can you make from the 4 tens shown in the diagram? (One group of 3 with 1 ten left.)

How can you make groups of 3 ones from 1 ten and 8 ones? (Change 1 ten to 10 ones to make $10+8$, or 18 ones.)



How many groups of 3 ones can you make from 18 ones? (Six groups of 3.)

Help children understand that when they divide tens, they get tens; when they divide ones, they get ones. This understanding helps with the placement of quotient figures.

Method B

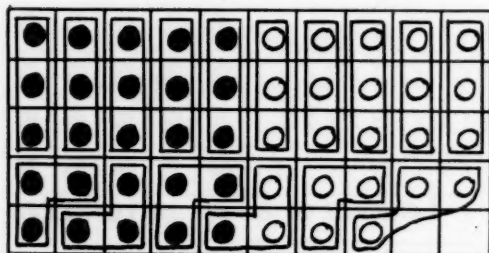
Method B is illustrated with the same example, $3\sqrt{48}$.

$$\begin{array}{r}
 16 \text{ (10+6)} \\
 3 \overline{)48} \\
 (10 \times 3) \rightarrow 30 \quad 10 \\
 \underline{18} \\
 (6 \times 3) \rightarrow 18 \quad 6
 \end{array}$$

10 groups of 3 = 30

6 groups of 3 = 18

16 groups of 3 = 48

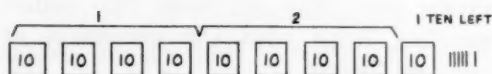


Are there as many as ten groups of 3 in 48?
If so, you can take out ten 3's at once.
How many are left? (18)
How many groups of 3 = 18? (6)
So, 10 + 6 or 16 threes = 48.
The answer may be totaled above the example after the work is completed.

Method A

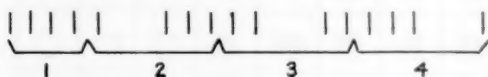
Method A is again illustrated, this time with the example $4\sqrt{96}$.

$$\begin{array}{r}
 24 \\
 4 \overline{)96} \\
 80 \text{ (2 tens } \times 4 \text{ or } 80) \\
 \underline{16} \text{ (4 ones } \times 4 \text{ or } 16) \\
 16
 \end{array}$$



How many groups of 4 can you make from 9 tens? (2 with 1 ten left)

How can you make groups of 4 from 1 ten and 6 ones? (Change 1 ten to ten ones to make 16 ones.)



How many groups of 4 can you make from 16 ones? (4 groups of 4.)

Method B

Method B is again illustrated, using the example $4\sqrt{96}$.

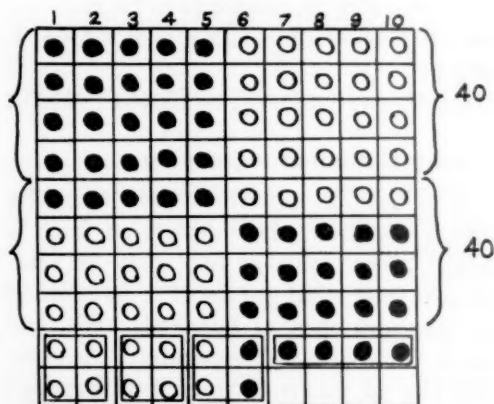
$$\begin{array}{r}
 24 \text{ (10+10+4)} \\
 4 \overline{)96} \\
 (10 \times 4) \rightarrow 40 \quad 10 \\
 \underline{56} \\
 (10 \times 4) \rightarrow 40 \quad 10 \\
 \underline{16} \\
 (4 \times 4) \rightarrow 16 \quad 4
 \end{array}$$

10 groups of 4 = 40

10 groups of 4 = 40

4 groups of 4 = 16

24 groups of 4 = 96

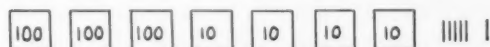


Can you make 10 groups of 4 from 96?
Can you make 10 more groups of 4 from what is left?

How many groups of 4 = 16?

Method A

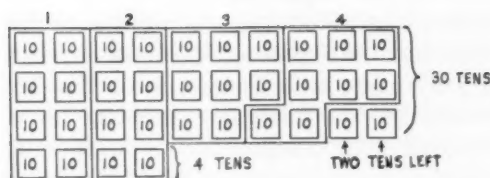
Method A is illustrated with a three-place dividend in the example $8\overline{)346}$.



How many groups of 8 can you make from 3 hundreds?

(Help children to see there are no groups of 8 until the hundreds are changed to tens.)

$$\begin{array}{r} 43 \\ 8 \overline{)346} \\ \underline{32} \\ 26 \\ \underline{24} \\ 2 \text{ R} \end{array}$$



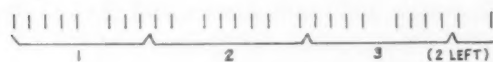
30 tens + 4 tens = 34 tens

How many groups of 8 = 34?

(4 groups of 8 and 2 left.)

34 tens = 4 groups of 8 tens and 2 tens left.

How many groups of eight can you make from 26 ones?



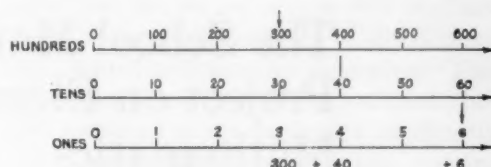
Three groups of 8 with 2 left.

Method B

Method B is illustrated with the same three-place dividend.

$$\begin{array}{r} 43 \quad (40+3) \\ 8 \overline{)346} \\ \underline{(40 \times 8) \rightarrow 320} \quad 40 \\ 26 \\ \underline{(3 \times 8) \rightarrow 24} \quad 3 \\ 2 \text{ R} \end{array}$$

Use the three number lines and your imagination to help you determine about how many groups of 8 you could make from 346.



Could you make 10 groups of 8 from 346? (Yes. $10 \times 8 = 80$)

How about 20 groups of 8? (Yes. $20 \times 8 = 160$)

How about twice 20? (Yes. $40 \times 8 = 320$)

How many groups of 8 = 26? (3 with 2 left)

Summary

At best the long division algorithm is difficult for children to learn. Regardless of the method used, they should be helped to reason out why each step in the procedure is necessary. Basic to success is keeping in mind the total job to be accomplished which is to regroup a given amount into groups represented by the divisor. Help the children focus on what is done next and why in order to achieve their goal. This emphasis will help them discover the steps to be taken and the order of steps.

You may wonder if children themselves can draw long division diagrams similar to those presented. They will have little difficulty with the diagrams for Method B which are similar to those recommended for learning multiplication facts. If the diagrams for Method A prove too complicated for some children, they can still derive much understanding from observing your demonstrations and participating in a discussion of what is happening and why.

The School Mathematics Study Group Project on Elementary-School Mathematics

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The work of the School Mathematics Study Group (MSG) is well known. Reports of its objectives, its organization, and its scope of activity have appeared in various sources.¹ This present account relates to one of the Study Group's more recently organized activities, the MSG Project on Elementary-School Mathematics.

Origin and organization of the project

The following statement by Professor E. G. Begle, Director of the School Mathematics Study Group, provides helpful background for this report of the project's origin, organization, and present status:

"It is the objective of the School Mathematics Study Group to bring about an improvement in the teaching of mathematics in our schools. The increasing dependence of our society on science and technology makes this improvement imperative. The number of our citizens skilled in mathematics must be greatly increased and an understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship.

"In trying to bring about an improvement in school mathematics, it might seem

at first glance that it would be best to start at the beginning, kindergarten, and to work up. However, our procedure has been just the opposite of this. We have set a goal to aim at, a decision as to what we would like to have our students know when they finish high school. For example, our goal for the college-capable student is a program in high school which will enable him to take in his first college year a substantial course in calculus. For the less capable student, our goal is a mastery and an understanding of as many mathematical skills as the student is capable of, and at the same time an understanding of the nature of mathematics and of the role of mathematics in our society.

"Having set our goals, we then work backwards. One of the School Mathematics Study Group projects which is currently under way is the design of courses for grades 9 through 12. These will not only provide the mathematical prerequisites for a formal course in calculus but also as much informal and intuitive background for such a course as possible.

"Another of our projects is concerned with grades 7 and 8. Here we are attempting to recast the program for these two years so that they will provide a much better intuitive background for the work of the higher grades.

"With the work on the curriculum for grades 7 through 12 well under way, it became appropriate to consider the mathematics curriculum in the elementary school. The high school mathematics curriculum rests on the foundation built in the first six grades, and an improvement in these grades might allow a substantial strengthening of the program for the higher grades.

"For this reason, a conference on elementary-school mathematics was held at the Conrad Hilton Hotel in Chicago on February 13 and 14, 1959. The [54] participants included college and university mathematicians, high school teachers, educational experts with special interest in arithmetic, supervisors, elementary-school teachers, psychologists, and representatives of scientific and government organizations having an interest in mathematics."²

The suggestions and recommendations growing out of this conference were referred to an ad hoc committee and translated into specific action proposals. In due time, the SMSG policy-making Advisory Committee approved the recommended downward extension of the Study Group's over-all work to include the elementary-school level, the National Science Foundation appropriated funds necessary to implement this new phase of SMSG activity, and a Panel on Elementary-School Mathematics was appointed to initiate and supervise the work of the new project.³ The members of this panel met for the first time on January 30, 1960—within a year after the conference on elementary-school mathematics gave impetus to the extension of SMSG activity to embrace grades K-6.

National Council co-operation

Readers of *THE ARITHMETIC TEACHER* will be interested to learn of the facilitating role played by The National Council of Teachers of Mathematics in establishing the SMSG project on elementary-school

mathematics. In December, 1958, the Council's Elementary-School Curriculum Committee (ESCC) submitted a report to the NCTM Board of Directors in which a preliminary plan of action was outlined for a comprehensive study of the elementary-school mathematics curriculum.⁴ This report included the recommendation that the proposed study be tied in as directly as possible with an appropriate ongoing mathematics curriculum project at the secondary-school level.

Action soon followed. The Board approved the ESCC report and authorized the president of the Council and the chairman of ESCC to explore with the director of the School Mathematics Study Group the possibility of Council co-operation in organizing a downward extension of SMSG's work to include activities at the elementary-school level along the general lines proposed in the ESCC report. As a consequence of this fruitful exploration, the director of the School Mathematics Study Group actively involved the authorized NCTM representatives in consultation at strategic times during the course of developments that culminated in the SMSG project on elementary-school mathematics.

Putting the program together

At the first meeting of the panel, mentioned earlier in this account, various circumstances and reasons prompted the decision to center project activity at this time on the mathematics program for grades 4, 5, and 6. In March, 1960, the members of the panel, augmented by five other persons, worked together uninterruptedly for a full week preparing a detailed outline of a suggested mathematics program for the intermediate grades (4-6) that would be in keeping with the broad objectives and specific ongoing projects of the School Mathematics Study Group.

For eight weeks during the summer of 1960, more than twenty persons worked

together intensively as a writing team on the Stanford University campus to translate the above outline into appropriate instructional material for both pupils and teachers. Fully half of the members of the writing team were classroom teachers or supervisors—persons directly and intimately associated with elementary schools. The other members of the writing team included some of the persons who had prepared the program outline for grades 4–6, along with additional mathematicians and mathematics educators who have a special interest in the mathematics curriculum at the elementary-school level.

Two small classes of children—one of beginning fourth-graders, the other of beginning sixth-graders—met daily during most of the eight-week period for mathematics instruction with a competent elementary-school teacher. These classes permitted members of the writing team to have portions of their material tried out immediately as it was written, thus making it possible to settle some crucial questions on an empirical basis.

The program as it has now evolved

In its present experimental form, the SMSG mathematics program for grades 4–6 is divided into 25 units, as follows:

Grade 4

- Concept of sets
- Numeration [including nondecimal bases]
- Introducing fractional numbers ["common fractions"]
- Properties of addition and subtraction of whole numbers
- Techniques of addition and subtraction of whole numbers
- Properties of multiplication and division of whole numbers
- Techniques of multiplication and division of whole numbers
- Sets of points
- Recognition of common geometric figures
- Linear measurement

Grade 5

- Factors, primes, and common denominators
- Properties and techniques of addition and subtraction of fractional numbers ["common fractions"]
- Extending systems of numeration ["decimal fractions," including addition and subtraction]
- *Multiplication using the decimal system of numeration [whole numbers and "decimal fractions"]
- *Division using the decimal system of numeration [whole numbers and "decimal fractions"]
- Side and angle relationships of triangles
- Measurement of angles
- Area

Grade 6

- Introducing the integers [the whole numbers and their negatives]
- Introducing exponents
- Multiplication and division of fractional numbers ["common fractions"]
- *Co-ordinates
- *Ratio applications, graphs, central tendency
- Sets and circles
- *Volume

All units except the five that are starred (*) are in completed preliminary form for restricted experimental use during the 1960–61 school year.⁵ The units have been listed in relation to the grade levels for which they are primarily designed. Some units, however, are being used this year at levels other than the designated one to provide content background that otherwise would not be developed.

The listing also has been organized to group together related units at each grade level. This organization is somewhat different from the sequence in which the units actually are being used in the classroom. In some instances more than one sequence of units is being tried out for experimental purposes with different classes.

New emphasis upon mathematical structure

Even though a mere list of unit titles has obvious shortcomings in conveying program content, several things should be rather apparent. In particular, the reader is very likely to sense the emphasis upon mathematical structure, the inclusion of a substantial amount of content from geometry, and an extension of arithmetic content to embrace topics normally not included at the intermediate-grades level. These things, coupled with others not evident from the unit listing, reflect the mathematical spirit which permeates the whole of the SMSG effort at all grade levels.

During the 1960-61 school year, use of the SMSG mathematics program for grades 4-6 is concentrated in eight experimental centers located from coast to coast. Each of these centers generally includes twelve classrooms—six in grade 4, three in grade 5, and three in grade 6. In keeping with SMSG policy, closely associated with each center is a college or university faculty member qualified to serve as a mathematics consultant to the elementary-school teachers who are using the experimental units. Similar consultant service is provided in the case of the twenty additional "points" which serve to augment the work of the eight experimental centers.

All in all, the plan of development and experimentation being carried out in connection with the SMSG mathematics program for grades 4-6 will follow the same general pattern as that used in connection with the SMSG programs for grades 7-8 and 9-12. At appropriate times in the future, progress reports relating to the School Mathematics Study Group project on elementary-school mathematics will ap-

pear in the SMSG Newsletter⁶ and in this section of THE ARITHMETIC TEACHER.

Notes

1. Two of the more recent accounts of the over-all program of SMSG activity appeared in the *School Mathematics Study Group Newsletter* No. 4 (March, 1960) and in the October, 1960 issue of *The Mathematics Teacher*.

2. Quoted from the SMSG *Report of a Conference on Elementary-School Mathematics*.

3. The members of the SMSG Panel on Elementary-School Mathematics are: E. G. Begle (Yale University), *ex officio*; E. Glenadine Gibb (Iowa State Teachers College), W. T. Guy (University of Texas), S. B. Jackson (University of Maryland), Irene Sauble (Detroit Public Schools), M. H. Stone (University of Chicago), and J. F. Weaver (Boston University).

4. A brief report of the work of the NCTM Elementary-School Curriculum Committee appeared in the March, 1959 issue of THE ARITHMETIC TEACHER and in the April, 1959 issue of *The Mathematics Teacher*.

5. Each unit consists of two parts: a booklet for the pupil and an accompanying commentary for the teacher. It is expected that the five starred units will be completed during the summer of 1961. For the present school year of 1960-61, material from the regular arithmetic program will be used in place of these units.

6. Interested persons wishing to receive future issues of the SMSG Newsletter may request to have their names placed on the mailing list by writing to Dr. John Wagner, Assistant to the Director, School Mathematics Study Group, Drawer 2502 A, Yale Station, New Haven, Connecticut.

Books and materials

Edited by

CLARENCE ETHEL HARDGROVE

Northern Illinois University, DeKalb, Illinois

The World Book Encyclopedia, 20 Volumes.

J. Morris Jones (editor in chief). Chicago: Field Enterprises Educational Corporation, 1960. Cloth, 11,720 pp. Cloth, \$139; to schools, \$104. Fabricoid binding, \$159; to schools, \$115.

This review is limited to the material on arithmetic in the *World Book Encyclopedia*. Other aspects of mathematics, such as algebra, calculus, and topology, are not discussed here since they are of primary interest in secondary school and college. Such topics, by the way, appear to have received good coverage.

To most people, the word "encyclopedia" conveys quite properly the impression of complete and detailed exhaustion of a subject. Perhaps, however, it is not until the reader endeavors to examine an encyclopedia for all references to a subject such as arithmetic that he truly can appreciate the word. The *World Book* is encyclopedic in its coverage of arithmetic. Four to six pages each are given to the large topics, "Arithmetic," "Addition," "Subtraction," "Multiplication," "Division," "Percentage," "Measurement," "Metric System," and "Decimal Number System." Ten pages are given to "Fraction." Less space, but more than mere mention, is given to "Ratio," "Proportion," "Number," "Roman Numerals," "Binary Arithmetic," "Zero," and similar topics. Far too numerous to mention are related topics such as "Computers,"

"Abacus," "Auditor," "Discount," "Weights," and "Measures."

The last volume of the set, Volume 20, is entitled *Reading and Study Guide*. It is an index, yet more too. It begins with a brief section on how to use the *World Book* for browsing, systematic study, and personal guidance. Then the remainder of the volume presents the contents of the entire set in topical outline form. Arithmetic is well represented here.

The general section, "Arithmetic," sets forth very well the value of arithmetic as a tool subject. It does not, however, call attention to the important aspects of the structure of arithmetic that contribute through intuitive understanding to a good foundation for more advanced mathematics. An analysis of arithmetic problems into two classifications, those solved by counting and those solved by measuring or comparing, seems too restricted. The main part of "Arithmetic" briefly outlines ways to work with numbers, both whole numbers and fractions. Here the development is good. Operations are clearly related to place value and the concept of base ten. Illustrative examples are both practical and well suited to their use as concrete beginnings for later abstractions.

A moderately serious weakness of "Arithmetic" is a confusion about just what are *numeral systems* and *number systems*, as seen in the following quotations from Volume A. On page 550, we read,

"Ways of putting numbers in order are called numeral systems, or sometimes *number systems*." But on page 552, we find, "Thus, in problems that relate to numbers of men, eggs, houses, and so on, we can answer in whole numbers. The numeral system 0, 1, 2, 3, and so on, fits this sort of problem and we have no need of fractions." Clearly the purpose of the second quotation is to contrast whole numbers and fractions, so the reference is to *number systems*, not about ways to write numbers.

At the risk of appearing too faithful to current movements to "clean up the language," I must add that in several places the *World Book* fails to be clear on this point. From "Decimal Number System [sic]," we learn that, "The English-speaking countries continue to use some measurements that are not based on the *decimal number system*." [Italics mine.] (Vol. D, p. 62.) Does the writer not mean "... on the base ten"?

From "Duodecimal," one finds that, "Duodecimal is the name given to a system of numbers based on 12." (Vol. D, p. 310.) In "Roman Numerals," there is the statement that, "All Roman numbers are written from seven basic numerals." (Vol. R, p. 395.) In "Binary Arithmetic," we note that, "Binary arithmetic uses an extremely simple number system." (Vol. B, p. 239.)

But in "Computers," the following quotation illustrates one way to express the above correctly: "Binary arithmetic is based on a numbering system that uses only two digits: 0 and 1." (Vol. C, p. 743.) Finally, in "Number," we are told, "Number is a word that tells *how many*." (Vol. N, p. 449.)

Here, it seems, the *World Book* lost an excellent opportunity to come as close as such a nontechnical publication can to clarifying the distinction between "number" and "numeral" and bringing to light the abstract nature of number.

Four important sections, "Addition," "Subtraction," "Multiplication," and "Division," are all organized similarly.

The following outline for "Addition" illustrates this:

- I. Learning to Hold
- II. Adding Larger Numbers
- III. Checking Addition
- IV. Addition Rules to Remember
- V. Fun with Addition
- VI. Practice Examples

Addition facts are illustrated carefully, starting with concrete counting situations. Where the importance of learning addition facts is stressed, meaningful interrelationships are pointed out as aids to learning the facts. The concept of base ten and place value are clearly shown to be the basis for addition beyond the facts.

"Subtraction" follows the same good pattern: flowing smoothly from concrete to abstract; careful attention to base ten and place value; the different problem situations using subtraction. Both "Addition" and "Subtraction" fail slightly in their practice examples, for there are no verbal problem situations given.

In "Multiplication," ideas are presented slightly more abstractly than in the sections just discussed. This seems quite proper. However, this may have led to a less meaningful approach to multiplying by even tens, hundreds, etc. The same comment applies where multiplying by a factor with an "inner zero" is shown.

The "Division" section relies almost exclusively on the notion of division as repeated subtraction. This, together with a strong emphasis on place value, is used to develop long division. This approach illustrates clearly the process of long division for the beginner. Examples are given showing that what you do with a remainder depends on the kind of problem.

In its development of division with decimal fractions, of short division, of checking division, etc., this article shares with the other major articles on arithmetic a concern for developing rules meaningfully.

The "Fraction" section has pictorial illustrations that are mathematically sound and that convey meanings both in developing basic concepts and in present-

ing operations with fractions. Practical applications are well used in illustrating the ideas in the section.

The section entitled "Percentage" is a fairly standard presentation of the subject. A wide range of genuine applications of percentage is exhibited in the development of the topic. However, the applications are not used in the practice examples. This section seems quite deficient in one aspect. No mention is made of the relation of per cent to ratio or to the use of proportions in calculations with per cents.

In the article on "Ratio," a ratio is defined as a "quotient of two numbers of the same kind." The context of what follows makes this clear. "There is no such thing as the ratio of two numbers unless the two numbers can be expressed in the same units." (Vol. R, p. 143.) This seems too narrow an interpretation. No mention, even in cross reference, is made to proportions or to per cent, an important instance of ratio.

In "Proportion," the presentation is abstract, i.e., without any application in illustration or practice example to problem-solving, as was done in the *World Book* presentation of other major areas of arithmetic. Reference here is made to ratio, but no connection with per cent is made.

In most of the arithmetic sections of any length, historical material is included and is, on the whole, quite good. One may regret the omission, in "Abacus," of reference to the conflict between abacists and algorists. In "Roman Numerals," three clever pictures use considerable space to show how awkwardly a Roman soldier would have performed multiplications with numbers expressed in Roman numerals. (Vol. R, p. 395.) There follows an algorithm similar to today's. However, was this ever an issue? A person experienced with computation on an abacus was (and still would be) rapid and accurate.

The articles on arithmetic have almost all been written by different persons. Most of the contributors are recognized authori-

ties on arithmetic. In the main, the writers of the articles have done excellent work in presenting their ideas completely, concisely, and meaningfully. Behind these articles lies a fairly uniform philosophy in the teaching of arithmetic. Undoubtedly this is due to the care given the editorial supervision. In spite of such care, certain fine points of mathematical correctness consisting of terminology, interrelation of topics, and pedagogy were missed.

The strong points of the *World Book* in its presentation of arithmetic far outweigh its weaknesses. Its articles have utilized, for the most part, the best developments of arithmetic textbook writing. Pictures are clear and up-to-date. Language is readable and up-to-date. Children like to read the *World Book*. (There were several times during this reviewing when minor conflicts arose because my two daughters were also using the volumes.) When they want to find a brief, yet clear and meaningful explanation of something in arithmetic, I will feel my children are safe in using the *World Book Encyclopedia*.

LYMAN C. PECK

Ohio Wesleyan University
Delaware, Ohio

Books received

Arithmetic, Fred Marer, Samuel Skolnik, and Orda E. Lewis. Boston: Little, Brown and Company, 1960. Cloth, viii+246 pp., \$4.00.

Arithmetic Handbook, Martin H. Ivener. Box 321, North Hollywood, California: Martin Publishing Company, 1960. Paper, 60 pp., \$1.00.

Elementary and Junior High School Mathematics Library, Clarence Ethel Hardgrove. Washington, D.C.: The National Council of Teachers of Mathematics, 1960. Paper, 32 pp., \$0.35.

High School Mathematics Library, William L. Schaaf. Washington, D.C.: The National Council of Teachers of Mathematics, 1960. Paper, 36 pp., \$0.40.

Magic House of Numbers, Irving Adler. New York: The John Day Company, 1957. Cloth, 128 pp., \$3.00.

Numbers Old and New, Irving and Ruth Adler. New York: The John Day Company, 1960. Cloth, 48 pp., \$2.00.

Report of the Nominating Committee

The Committee on Nominations and Elections presents its nominees for offices to be filled in the 1961 election. The term of office for the two vice-presidents is two years. Three directors are to be elected for terms of three years.

In making nominations for the three director positions, the Committee followed the directive adopted by the Board of Directors in 1955 which states, "Nominations shall be made so that there shall be not more than one director elected from each state, and that there shall be one director, and not more than two, elected from each region." Members may consult *The Mathematics Teacher* for October, 1955, for a map of the regions as they are now defined.

Ballots will be mailed on or before February 15, 1961 from the Washington Office to members of record as of that date. Ballots returned and postmarked not later than March 15, 1961 will be counted.

The Committee wishes to thank the many members of the NCTM for help in

giving their suggestions for nominees. It is hoped that all members of our organization will be sure to exercise their privilege of voting.

OSCAR F. SCHAAF, *Chairman*
HAROLD FAWCETT
W. EUGENE FERGUSON
LENORE JOHN
HOUSTON T. KARNES
W. C. LOWRY
IDA B. PUETT
MAX SOBEL
LOTTCHEN HUNTER

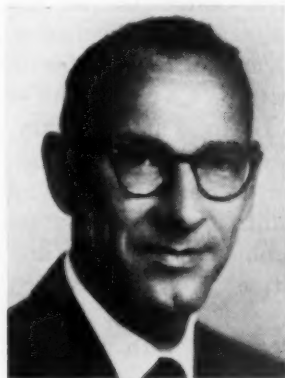
Nominees: vice-president, college level

Bruce E. Meserve

Professor of Mathematics and Chairman, Department of Mathematics, Montclair State College, Upper Montclair, New Jersey.



Bruce E. Meserve



Myron F. Rosskopf

A.B., Bates College; A.M. and Ph.D., Duke University.

Teacher of mathematics, Moses Brown School, 1938-41; graduate work at Duke University, 1941-42, 1945-46; Civilian Public Service, 1942-43; U.S. Army, 1943-45; instructor (1946-48), assistant professor (1948-54) of mathematics, University of Illinois; associate professor (1954-57), professor and chairman of mathematics (1957-), Montclair State College, teacher in Montclair State College Demonstration High School, 1954-57, 1958-59, 1960.

Fellow, AAAS. Member: AMS, MAA, AMTNE, AMTNYC, CASMT, NJAS, NJEA, Phi Beta Kappa, Sigma Xi, and Kappa Mu Epsilon. Vice-president of AMTNJ, 1960-; vice-president, Illinois Council of Teachers of Mathematics, 1952-54; chairman, New Jersey Section of MAA, 1957-58. Member: UICSM, 1951-54; NAS/NRC Film Evaluation Board, 1957; panel on teacher training materials of SMSG; panel on teacher training of the Committee on the Undergraduate Program in Mathematics.

Listed in: *American Men of Science*, *Leaders in American Science*, *Who's Who in American Education*, *Who's Who in the East*, and *World Directory of Mathematicians*.

Activities in NCTM: Member, Board of Directors, 1958-61; Budget Committee, 1959-62; Yearbook Planning Committee, 1957-61. Representative of NCTM on AAAS Co-operative Committee on the Teaching of Science and Mathematics, 1958-62. Past chairman of Committee on Research, Committee on Co-ordination of Mathematics with Business and Industry, Committee on Secondary School Standards. Past member of Secondary School Curriculum Committee, Committee on Institutes and Summer Workshops, and Editorial Committee of Twenty-third Yearbook.

Publications: Coauthor with Virgil S. Mallory and Kenneth C. Skeen of *First Course in Geometry*; *General Mathematics*

(2nd ed.); *First Course in Algebra*, and *Second Course in Algebra*. Coauthor with J. B. Rosenbach, E. A. Whitman, and P. M. Whitman of *College Algebra* (4th ed.); *Essentials of College Algebra* (2nd ed.), and *Intermediate Algebra for Colleges* (2nd ed.). Author of *Fundamental Concepts of Algebra*; *Fundamental Concepts of Geometry*, and over forty articles and reviews in professional journals.

Myron F. Rosskopf

Professor of Mathematics, Teachers College, Columbia University, New York, New York.

A.B., University of Minnesota; M.A., University of Minnesota; Ph.D., Brown University.

Teacher and head of department of mathematics, John Burroughs School, Clayton, Missouri; associate professor of mathematics and education, Syracuse University.

Member: NCTM, MAA, AMS, Sigma Xi, Phi Beta Kappa, Phi Delta Kappa, Pi Mu Epsilon; AAUP, New York Academy of Sciences.

Activities in NCTM: Associate editor, *The Mathematics Teacher*; past member and chairman, Yearbook Planning Committee; member of Nominations Committee, 1958; delegate to International Congress of Mathematicians, Amsterdam, The Netherlands, 1954.

Other activities: Charter member of Association of Mathematics Teachers of New York State; past president of AMTNYS; past member of council and executive committee of AMTNYS; member of writing groups of Commission on Mathematics, CEEB.

Publications: Numerous articles appearing in *The Mathematics Teacher*, *School Science and Mathematics*, *Teachers College Record*, and other periodicals; chapters in various NCTM yearbooks; coauthor of textbooks on elementary, secondary, and college levels.



Helen L. Garstens

Nominees: vice-president, junior high school level

Helen L. Garstens

Associate Director, University of Maryland Mathematics Project, University of Maryland; Lecturer in Mathematics and Education, University of Maryland.

B.A., Hunter College, New York City; graduate study at Columbia University, University of Virginia, Georgetown University.

Senior high school teacher in New York City; teacher in elementary, junior high, and senior high schools of Arlington, Virginia; supervisor of secondary mathematics, Arlington, Virginia; demonstration teacher for National Science Foundation Summer Institute for Junior High School Mathematics Teachers at University of Maryland.

Member: NCTM, Arlington Chapter of NCTM, AAAS, Washington Academy of Sciences, Pi Mu Epsilon, Phi Beta Kappa.

Activities in NCTM: Member; participant in NCTM convention programs; organizer of Arlington Chapter of NCTM; committee member for Washington, D.C. meeting.

Other activities: Extensive participation as consultant and lecturer in in-service activities for mathematics teachers sponsored by school systems, universities, individual chapters of NCTM; guest lec-



Mildred Keiffer

turer at summer institutes and workshops; Committee for Gifted in Mathematics, NEA; writing of curriculum materials for UMMaP; writing team for grades 4, 5, 6 for SMSG; citation by Washington Academy of Sciences for work in mathematics curriculum development and teacher training.

Publications: Articles in *THE ARITHMETIC TEACHER*, *The Mathematics Teacher*; "Experimental Mathematics for the Seventh Grade," *NEA Journal*; contributor to *Mathematics for the Junior High School*, *First Book*, *Second Book*, UMMaP.

Mildred Keiffer

Supervisor of Mathematics, Secondary Schools, Cincinnati, Ohio.

A.B., University of Cincinnati; M.A., Columbia Teachers College; workshops at Duke, Rutgers, University of Wisconsin, University of Maryland.

Teacher in elementary, junior, and senior high schools of Cincinnati; supervising teacher, assistant supervisor, supervisor of secondary school mathematics of the Cincinnati Public Schools.

Member: NCTM, MAA, AAAS, ASCD, CASMT; Ohio Council of Teachers of Mathematics; Mathematics Club of Greater Cincinnati; Phi Beta Kappa; Delta Kappa Gamma; Kappa Delta Pi; OEA; life member of NEA.

Activities in NCTM: Member at various times of NCTM committees on Sup-

plementary Publications, Membership, Nomination, Television; Editorial Board, *The Mathematics Teacher*; Subcommittee of the Secondary School Curriculum Committee; speaker at various meetings; chairman of NCTM Regional Orientation Conference in Cincinnati.

Other activities: Consultant for OCTM Workshops, Shell Program at Cornell; former president of Cincinnati Mathematics Club, Ohio Council of Teachers of Mathematics; SMSG writing teams at the University of Michigan and Stanford University; SMSG Panel for Non-College-Bound Students; Engineering Society Mathematics Tournament Committee.

Publications: Coauthor with Margaret Joseph of *Basic General Mathematics*; contributor to NCTM bulletins and pamphlets, and to *Bulletin of National Association of Secondary School Principals*.

Nominees for board of directors

Max Beberman

Central Region

Professor of Education, University of Illinois; Teacher of Mathematics, University of Illinois High School; Director, University of Illinois Committee on School Mathematics [UICSM] Project.



Max Beberman



Lurnice Begnaud



John A. Brown

B.S., College of the City of New York; A.M. and Ed.D., Columbia University.

Teacher of mathematics in Nome (Alaska) public school; Teachers College, Riverdale (New York) Country Day School; associate professor of education, Florida State University.

Member: NCTM, MAA, AMS, CASMT, AAAS, Phi Delta Kappa, Kappa Delta Pi, Pi Mu Epsilon.

Activities in NCTM: Editor, *The Mathematics Student Journal*, 1956-58; member of Publications Board.

Publications: various articles in *The Mathematics Teacher*, *The School Review*, *Educational Leadership*; author of *An Emerging Program in School Mathematics* (Inglis Lecture); coauthor of *Algebra, Course I; Algebra, Course II; High School Mathematics* [Units 1, 2, 3, 4, 5, 6].

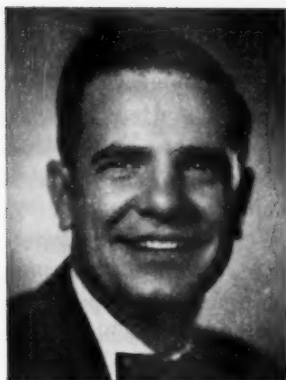
Lurnice Begnaud

Southwestern Region

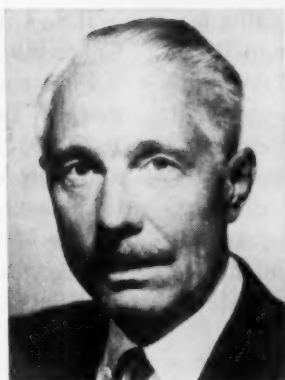
Supervising Teacher, Southwestern Louisiana Institute, Lafayette, Louisiana; Head of Mathematics Department, Lafayette Senior High School, Lafayette, Louisiana.

B.S., Southwestern Louisiana Institute; M.A., Louisiana State University.

Mathematics teacher, Milton High School, Milton, Louisiana; Carenero High School, Carenero, Louisiana; staff member of Louisiana State University Summer



William T. Guy, Jr.



Julius H. Hlavaty



Rachel P. Keniston

Mathematics Institute, 1950-59; staff member of Duke University Summer Mathematics Institute, 1952.

Member: NCTM, Phi Kappa Phi, Kappa Mu Epsilon, NEA, Louisiana Teachers Association, Association for Student Teaching, American Association of University Women.

Activities in NCTM: state representative for NCTM in Louisiana; past president of Louisiana-Mississippi Section of NCTM; appearances on convention programs.

Other Activities: Consultant in numerous summer parish (county) workshops throughout Louisiana; participant in convention programs on a local, state, and national level; past president, Mathematics Section of Louisiana Teachers Association.

John A. Brown

Northeastern Region

Professor of Education and Mathematics, University of Delaware, Newark, Delaware.

B.S., LaCrosse State College; M.S., Ph.D., University of Wisconsin.

Teacher at Mattoon High School, Mattoon, Wisconsin; Oconto High School, Oconto, Wisconsin; Merrill High School, Merrill, Wisconsin; Wisconsin High School, University of Wisconsin; State

University Teachers College, Oneonta, New York.

Member: NCTM, AAAS, AERA, NEA, MAA Delaware Mathematics Council.

Activities in NCTM: former departmental coeditor; member of Committee on Guidance Pamphlet.

Other activities: member of SMSG junior high panel; SMSG writing team, 1958 and 1959; director of SMSG Center at the University of Delaware; departmental coeditor for *The American Mathematical Monthly*; regional representative of MAA Visiting Lecture Program.

Publications: Contributor to NCTM Twenty-second Yearbook; coauthor of textbooks; various magazine articles.

William T. Guy, Jr.

Southwestern Region

Professor and Chairman, Department of Mathematics, University of Texas, Austin, Texas.

B.S., A&M College of Texas; M.A., University of Texas; Ph.D., California Institute of Technology.

Instructor, Department of Applied Mathematics, University of Texas; student assistant and institute scholar, California Institute of Technology; assistant professor, professor, Department of Mathematics, University of Texas. Guest lecturer for several NSF summer institutes

for secondary teachers; MAA visiting lecturer for high schools, Colorado-Wyoming, 1959, and Illinois, 1960. Mathematics teacher in several academic-year and summer institutes held at University of Texas; director, 1960 Summer Institute for Elementary School Personnel; director, 1959-60 Visiting Lecturer Program of Mathematics for the Texas Academy of Science; member of Commission on Mathematics, Texas Education Agency.

Member: NCTM, MAA, AMS, Texas CTM; Fellow, Texas Academy of Science; Fellow, AAAS; Sigma Xi, Tau Beta Pi, Sigma Pi Sigma.

Activities in NCTM: several invitational addresses to national and regional meetings; refereed many papers for *The Mathematics Teacher*.

Other activities: governor, Mathematical Association of America; past chairman, Texas Section, MAA; consultant to NSF-sponsored mathematics teacher institutes; member, MAA Committee on Undergraduate Program in Mathematics, Committee on Summer Institutes; AAAS representative at two TEPS and NASDTEC conferences; member, SMSG panel and summer writing project in mathematics for elementary school; consulting mathematics editor for a textbook publisher.

Listed in: *American Men of Science*, *Leaders in American Science*, and *Who's Who in Texas*.

Publications: Contributor to *Mathematics Monthly*, *Bulletin of the American Mathematical Society*, *Journal of the Texas Academy of Science*, *Proceedings of the sixth Midwestern Conference on Fluid Mechanics*.

Julius H. Hlavaty

Northeastern Region

Chairman, Department of Mathematics, DeWitt Clinton High School; Instructor, Teachers College, Columbia University.

B.S., City College of New York; Ph.D., Columbia University.

First chairman, Department of Mathematics, Bronx High School of Science, 1938-53; National Science Foundation and New York State In-Service Institutes for Teachers of Mathematics; fellow, Rocky Mountain Workshop, Commission on Secondary School Curriculum, summer 1938; staff, Curriculum Workshop, Syracuse University, summers 1939, 1940; teacher, Evander Childs High School, New York City, 1930-38; Theodore Roosevelt High School, New York City, 1929-30.

Member: Phi Beta Kappa; NCTM, MAA, Central Association of Teachers of Science and Mathematics; Mathematics Chairmen's Association of New York City; Association of Teachers of Mathematics of New York City.

Activities in NCTM: Chairman, Committee on Mathematics for the Talented, 1958—; speaker and panel chairman at national conventions; member, Conference on Directions, 1959; chairman, Planning Committee for Yearbook for the Talented.

Other activities: Director, Mathematics Commission Program, College Entrance Examination Board; School Mathematics Study Group—advisory committee, executive committee, monograph panel, editorial board, chairman of Committee to Evaluate Sample Textbooks, chairman of Committee on the Gifted; NEA Invitational Conference on the Academically Talented, Washington, D.C., 1958; consultant, University of Illinois Committee on School Mathematics; Mathematics committee, School and College Study of Admission with Advanced Standing, 1952-55; test consultant, War Department AGO, 1944-46; mathematics committee, Commission on Secondary School Curriculum (the 8-year study); chairman of Mathematics Sub-Committee of the NEA Project on the Academically Talented Pupil; past president, Mathematics Chairmen's Association and of Association of Teachers of Mathematics, New York City;

past chairman, Standing Committee on Mathematics, Board of Education, New York City; speaker at numerous in-service training and summer institutes (NSF); consultant for school systems at Yardley, Pennsylvania, Ridgewood and Hackensack, New Jersey; New Rochelle, New York; Lexington, Massachusetts; Los Angeles and San Diego, California; Advisory Conference on Co-ordination of Curriculum Studies (NSF), 1960; consultant, Metropolitan School Study Council.

Publications: Contributor to *The Mathematics Teacher*, *College Board Review*; translator, Felix, *The Modern Aspect of Mathematics*; editor, *Mathematics for the Academically Talented Student*; contributor, *Program Provisions for the Mathematically Gifted Student*; author, *Review Digest in Solid Geometry*.

Rachel P. Keniston

Western Region

Teacher of Mathematics and Chairman of the Mathematics Department at A. A. Stagg Senior High School, Stockton, California.

A.B., Smith College, Northampton, Massachusetts; M.A., College of Pacific, Stockton, California.

Mathematics teacher in Contoocook, New Hampshire, Enfield, Connecticut, Castilleja, Palo Alto, California, and currently in Stockton Unified School District, Stockton, California.

Member: NCTM, NEA, California Mathematics Council, New England Association for Teachers of Mathematics, California Teachers Association, Phi Kappa Phi, Delta Kappa Gamma.

Activities: Former vice-president of California Mathematics Council; former county representative of California Mathematics Council; former member of NCTM Committee on Supplementary Publications; speaker at numerous mathematics meetings at both state and national levels; instructor for a week's course at the summer meeting of NEATM, 1955, chairman of local mathematics curriculum committee, 1956-59.

Publications: Coauthor of textbooks, *Plane Geometry* and *High School Geometry*; article in Eighteenth Yearbook of NCTM; articles in *Bulletin of the California Mathematics Council*.

Professional dates

The information below gives the name, date, and place of meeting with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE ARITHMETIC TEACHER. Announcements for publication should be sent at least ten weeks early to the Executive Secretary, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C.

NCTM convention dates

Thirty-ninth Annual Meeting

April 5-8, 1961

Conrad Hilton Hotel, Chicago, Illinois

Robert Sisler, Morton High School West, 2400 Home Avenue, Berwyn, Illinois

Joint Meeting with NEA

June 28, 1961

Atlantic City, New Jersey

M. H. Ahrendt, 1201 Sixteenth Street, N.W., Washington 6, D.C.

Twenty-first Summer Meeting

August 21-23, 1961

University of Toronto, Toronto, Canada

Father John C. Egsgard, St. Michael's College School, 1515 Bathurst Street, Toronto 10, Canada

Other professional dates

The Greater Cleveland Council of Teachers of Mathematics

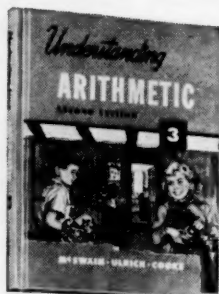
February 16, 1961

Roehm Junior High School, Berea, Ohio

Bessie Kisner, Strongsville High School, Strongsville, Ohio

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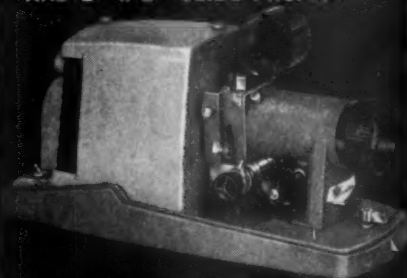
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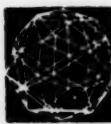
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